A Mathematical Modelling on Two Phase Arterial Blood Flow During Breast Cancer

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Abstract: In this present research paper, to the mathematical studies in two phase blood circulation during Brest cancer. The two phases of blood are supposed as first plasma phase and second that of red blood cells. The extended non-Newtonian power law has been used for the formulation of the equation of continuity and the equation of motion into tensorial form. The clinical data for blood pressure and haemoglobin has been collected for the breast cancer patient. The solution techniques adopted are analytical as well as numerical. The correlation of blood pressure drop v/s haematocrit has been obtained. The interpretation of graphical presentation is fruitful for diagnosis.

***Keywords*: - Two phase arterial blood flow, Breast Cancer, non- Newtonian, Extended Power law model, haematocrit, blood pressure drop.**

1. INTRODUCTION: -

The protein is present in the milk duct which forms a shaped lump in contact with the muscle. Where there is present the artery shaped lumped grade the haemoglobin**.** According to WHO, breast cancer has become a common cancer in the world. It is found in both men and women but it is mostly seen in women. And the mortality rate during breast cancer is highest in women. During 2020, 2.3 million women in the world suffered from this cancer and 685,000 died [1].

According to WHO, there have been 178361 new cases of breast cancer in India, which is 7.9% of the entire world, out of which 90408 have been found, which is 13% of the entire world[1]. In India, the incidence of breast cancer among women has been 13.2% and 10.25% have been found in India[2].

Breast cancer is a type of the cancer that begins as a growth of abnormal cells in the breast tissue. Haemoglobin (Hb) levels have been shown to be associated with treatment outcomes and survival in breast cancer patients [3].

1.1 Structure and function of systemic circulatory system:

Systemic circulation carries oxygenated blood from the left ventricle of heart, through the arteries, to the capillaries in the tissues of body.

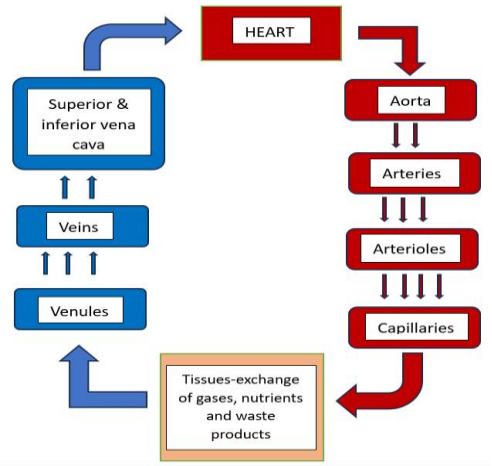

Fig.1: Systemic Circulatory System

1.2 Structure and function of arteries: Arteries are an important part of the systemic circulatory [4]. Systemic arteries carry oxygen-rich blood from the left ventricle of the heart to all parts of the body. Blood moves from large elastic arteries into smaller branches, eventually becoming tiny arterioles [5].

2. Real Model

A. Choice of frame of reference

In this context, according to V. Upadhyay and S.K. Chaturvedi we express the physical quantities related to blood flow in a tensor form, which provides a more realistic representation of the dynamics.

Let the coordinate axes be , where O is the origin and k = 1,2,3 denotes the three spatial dimensions. The state of the moving blood is described by functions that represent the distribution of the blood velocity = (,), where are the spatial coordinates and t is time[6].

B. Constitution of two-phase blood volume: In the two-phase blood flow model, blood is treated as a homogeneous mixture of plasma and RBCs. Plasma is considered the liquid phase, while RBCs are treated as liquid packets enclosed within a semi-permeable membrane, which can alter its shape and size. The volume of other formed elements is not considered in this model. The behavior of blood is almost Newtonian at high shear rates, at low shear rates blood pressure shows stress and non-Newtonian behavior. To flow blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells [7].

Let the volume fraction occupied by blood cells in a unit volume be denoted by Z, where Z = $\frac{\quad}{100}$ and Hct is the haematocrit (three times of haemoglobin[13]), representing the volume percentage of blood cells[8]. Consequently, the volume fraction occupied by plasma will be (1−Z).

If the mass ratio of cells to plasma is r, this can be expressed as:

$$r = \frac{Z\rho_s}{(1-Z)\rho_q}$$

where $\rho_s$and $\rho_q$are the densities of the blood cells and plasma, respectively.

C. Constitutive equation: Arterial blood shows a non-Newtonian nature. The constitutive equation of blood is:

$$T'^{ij} == -pg^{ij} + \mu_m(e^{ij})^n = -pg^{ij} + T''^{ij}, \quad \text{where } \mu_m = Z\mu_s + (1-Z)\mu_q \tag{2.1}$$

WhereT'^{ij} is stress tensor and $T''^{ij}$is shearing stress tensor. where T^{ij} , T''^{ij}, e^{ij}, g^{ij}, p, μ_m, μ_s and μ_q are stress tensor, shear stress, strain rate, conjugate metric tensor, pressure, viscosity of mixture (whole blood), core layer (RBC), and plasma layer, respectively.

D. Boundary conditions:

1. Velocity of blood flow on the axis of arteries at $= 0$ will be maximum and finite i.e. $v = \max$.
2. The velocity of the blood flow on the wall of artery at $r = R$, where R is the radius of the artery, will be zero. This condition is well known as a no-slip condition, $V = 0$ at $r = R$.

3. Mathematical Modelling/Formulation:

The equation of continuity for power law flow will be as follows:

$$\frac{1}{\sqrt{g(gv^i),i}} = 0 \tag{2.2}$$

Again, the equation of motion is extended as follows:

$$\rho_m \frac{\partial v^i}{} + \rho \; v^j v^i_{,} = T'^{,ij}{}_{,} \tag{2.3}$$

Where is taken from constitutive equation of power law flow.

$\rho_m = Z\rho_s + (1-Z)\rho_q$density of blood and $\mu_m = Z\rho_s + (1-Z)\rho_q$ is the viscosity of mixture of blood.

$$Z = \frac{}{100}$$

is volume portion of blood cells. Hct is haematocrit. Other symbols have their usual meanings. Since the blood vessels are cylindrical, the above governing equation have to be transformed into cylindrical co-ordinates. As we know earlier.

$$x^1 = r,\ x^2 = \theta, x^3 = z,$$

Matrix of the metric tensor in cylindrical coordinates is as follows:

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & {}^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

While the matrix of the conjugate metric tensor is as follows:

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/{}^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Whereas the Christoffel's symbols of 2nd kind are as follows:

$\{ {1 \atop 2\,2} \} = -r$, $\{ {2 \atop 2\,1} \} = \{ {2 \atop 1\,2} \} = \frac{1}{r}$, remaining other is zero[9,12].

The Relation between contravariant and physical components[9] of the velocity of blood flow will be as follows:

$$\sqrt{g_{11}v^1} = v_r \Rightarrow v_r = v^1$$

$$\sqrt{g_{22}v^2} = v_\theta \Rightarrow v_\theta = rv^2$$

$$\text{And } \sqrt{g_{33}v^3} = v_z \Rightarrow v_z = v^3$$

Again, the physical components of $-p_{,j}\, g^{ij}$ are $-\sqrt{g_{ij}}\, p_{,j} g^{ij}$

The matrix of physical components of shearing stress-tensor

$T'^{ij} = \mu\ (e^{ij})' = \mu\ (\quad + \quad)$ will be as follows:

$$\begin{bmatrix} 0 & 0 & \mu_m \left(\frac{}{dr}\right) \\ 0 & 0 & 0 \\ \mu_m \left(\frac{dv}{dr}\right)^n & 0 & 0 \end{bmatrix}$$

The covariant derivative T'^{ij} is

$$T'^{ij} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} \left(\sqrt{g T'^{ij}}\right) + \{ {i \atop } \} T'^{ij}$$

Keeping in view the above facts, the governing Tensorial equation can be transformed into a cylindrical form which is as follows:

1) Equation of Continuity

$$\frac{}{} = 0 \qquad (2.4)$$

2) Equation of Motion

a) r- component

$$-\frac{\partial p}{} = 0 \qquad (2.5)$$

b) θ – component

$$0 = 0 \qquad (2.6)$$

c) z- component

$$0 = -\frac{}{} + \frac{\mu_m}{} \frac{\partial}{} \left[\left(\frac{}{}\right) \right] \qquad (2.7)$$

Here this fact has been taken in view that the blood flow is axially symmetric in arteries concerned[7], i.e. $= 0$, , do not depend upon also the blood flows steadily, i.e.

$$\frac{}{} = \frac{}{} = \frac{\partial v_\theta}{} = \frac{}{} = 0$$

Solution: On integrating equation (2.4) we get

$$v_z = v(r) \text{ Because } v \text{ does not depend upon } \theta. \qquad (2.8)$$

The integration of the equation of motion (2.5) yields:

$$P = p(z), \text{ since p does not depend upon } \theta. \tag{2.9}$$

Now, with the help of equations (2.8) and (2.9), the equation (2.7) converts in the following form:

$$0 = -\frac{dp}{} + \frac{\mu_m}{}\frac{d}{}[r\,(\frac{dv}{})^n] \tag{2.10}$$

The pressure gradient $-(\frac{dp}{}) = p$ of blood flow in the arteries remote from heart can be supposed to be constant and hence the equation (2.10) takes the following from:

$$\frac{d}{}[r\,(\frac{dv}{})^n] = -\frac{pr}{\mu_m} \tag{2.11}$$

On integrating the equation (2.11), we get

$$r\,(\frac{dv}{})^n = -\frac{pr^2}{2\mu_m} + A \tag{2.12}$$

We know that the velocity of blood flow on the axis of cylindrical arteries is maximum and constant. So that we apply the boundary condition: at $r = 0, v = {}_0$(constant), on equation (2.12) to get the arbitrary constant $A = 0$. Hence the equation (2.12) takes the following form

$$(\_) = -\frac{pr^2}{2} \Rightarrow -\frac{dv}{} = (\frac{\_}{2})^{\frac{1}{}} \tag{2.13}$$

The equation (2.13) is integrating once again, we get

$$v = -(\frac{p}{2})^{\frac{1}{}}\frac{^{\frac{1}{}+1}}{(\frac{}{}+1)} + \tag{2.14}$$

To determine the arbitrary constant B, we apply the no-slip condition on the inner wall of the arteries: at $= ,\ = 0$, where = radius of vessels, on equation (2.14) so as to get

$$= (\frac{\_}{2})^{\frac{1}{}}\frac{^{\frac{1}{}+1}}{(\frac{}{}+1)})$$

Hence the equation (2.14) takes the following from:

$$= (\frac{\_}{2})^{\frac{1}{}}\frac{}{\frac{1}{}+1}(^{\frac{1}{}+1} - r_n^{\frac{1}{}+1}) \tag{2.15}$$

Equation (2.15) determines the velocity of blood flow in the arteries remote from the heart where P is a gradient of blood pressure and is the viscosity of blood mixture.

Bio-physical interpretation for artery blood vessel

The flow flux of blood through the arteries is-

$$Q = \int_0^R V.2\pi r dr = \int_0^R [\frac{P}{2\eta_m}]^{\frac{1}{}}\frac{n}{n+1}(R^{\frac{1}{}+1} - r^{\frac{1}{}+1})$$

$$Q = [\frac{P}{2\eta_m}]^{\frac{1}{}}\frac{n2\pi}{n+1}[\frac{R^{\frac{1}{}+1}.r^2}{2} - \frac{nr^{\frac{1}{}+3}}{3n+1}]_0^R$$

$$Q = [\frac{P}{2\eta_m}]^{\frac{1}{n}} \frac{n2\pi}{n+1} (\frac{(n+1).R^{\frac{1}{+3}}}{2(3n+1)})$$

$$Q = [\frac{P}{2\eta_m}]^{\frac{1}{}} \frac{n\pi R^{\frac{1}{n}+3}}{3n+1} \quad (2.16)$$

4. Result and Discussion

Patient Name: XYZ Age: 54

Table 1. Data between real clinically blood pressure drop (clinical data)

Date	**Haemoglobin (gram/dl)**	**Haematocrit $(3 \times HB)$ kg/l**	**Blood Pressure (mmHg)**	**BPD in Pascal Second $(\frac{+}{2}) - S$ (BPD*133.322)**
05/09/2024	14	0.42	120/70	3333.05
16/09/2024	11.3	0.339	110/90	1333.22
28/09/2024	11.2	0.336	110/60	3333.05
10/10/2024	11.7	0.351	140/90	3333.05
26/10/2024	12.3	0.369	130/80	3333.05

Source: Setha Cancer Hospital, Narmdapuram, (M.P.)

Observation: Haematocrit v/s blood pressure drop during breast cancer

In according to used clinical data (Haematocrit) Hct = 0.351 and Pressure drop$(P_f - P_i) =$ $3303.325 Pascalsecond.$

$$P(z) = \frac{P_f - P_i}{z_f - z_i}$$

Viscosity of mixture $= 0.035$. [10]

Viscosity of plasma $= 0.0015$. [11]

And by using relation $\mu_m = \mu_s Z + \mu_q(1 - Z)$ (2.17)

We get μ_s(Viscosity of RBCs)

$$\Rightarrow 0.035 = \mu_s(0.00351) + 0.0015(1 - 0.00351)$$

$$\Rightarrow 0.035 = \mu_s(0.00351) + 0.0015(0.99649)$$

$$\Rightarrow 0.035 = \mu_s(0.00351) + 0.001494735$$

$$\Rightarrow 0.033505265 = \mu_s(0.00351)$$

$\Rightarrow \mu_s = 9.5456595441595$Pascal second

Again using (2.17) relation and change in to the haematocrit-

$$= 9.545644544159 \quad + 0.0015$$

From equation (2.16)

$$P = --$$

We get

$$Q = [\frac{\Delta P}{2\eta_m \Delta_z}]^{\frac{1}{n}} \frac{n\pi R^{\frac{1}{n}+3}}{3n+1} \tag{2.18}$$

Where $= 1000$ ___ $= 0.0167 \frac{lit}{\_\_} = 1.67 \times 10^{-5} m^3/sec$

Length of systemic artery $\Delta_Z = 0.0347m$

Radius of cerebral artery =2.55 $= 0.00255$

Putting the values of Q, ΔP, Δ_z and R in equation (2.18)

$$0.0000167 = [\frac{3333.05}{2 \times 0.035 \times 0.0347}]^{\frac{1}{n}} \cdot \frac{3.14 \times n \times (0.0025)^{\frac{1}{n}+3}}{3 \quad + 1}$$

$$0.0000053184713 = [1372190.2017291]^{\frac{1}{n}} \frac{n \times (0.0025)^{\frac{1}{n}} \times (0.0025)^3}{3 \quad + 1}$$

$$0.0000053184713 = [1372190.2017291]^{\frac{1}{n}} \frac{n \times (0.0025)^{\frac{1}{n}} \times 0.000000015625}{3 \quad + 1}$$

$$340.3821632 = [3430.4755043227]^{\frac{1}{n}} \times \frac{}{3 \quad + 1}$$

We get the value of n

$$= 1.133$$

$$Q = [\frac{\Delta P}{2\eta_m \Delta_z}]^{\frac{1}{n}} \frac{n\pi R^{\frac{1}{n}+3}}{3n+1}$$

$$0.0000167 = [\frac{\Delta P}{2 \times \mu_m \times 0.0347}]^{\frac{1}{1.133}} \cdot \frac{3.14 \times 1.133 \times (0.0025)^{\frac{1}{1.133}+3}}{(3 \times 1.133) + 1}$$

$$0.0000167 = [\frac{\Delta P}{2 \times \mu_m \times 0.0347}]^{\frac{1}{1.133}} \cdot \frac{3.55762 \times (0.0025)^{\frac{1}{1.133}+3}}{4.399}$$

$$0.0000167 = [\frac{\Delta P}{2 \times \mu_m \times 0.0347}]^{\frac{1}{1.133}} \cdot \frac{3.55762 \times (0.0025)^{\frac{1}{1.133}} \times (0.0025)^3}{4.399}$$

$$0.0000167 = [\frac{\Delta P}{2 \times \mu_m \times 0.0347}]^{\frac{1}{1.133}} \cdot \frac{3.55762 \times (0.0025)^{\frac{1}{1.133}} \times 0.000000015625}{4.399}$$

$$0.0000167 = [\frac{\Delta P(0.0025)}{2 \times \mu_m \times 0.0347}]^{\frac{1}{1.133}} \cdot \frac{0.0000000555878}{4.399}$$

$$1321.57235940260272937587 = [\frac{\Delta P(0.0025)}{2 \times \mu_m \times 0.0347}]^{\frac{1}{1.133}}$$

$$3437.13066730297547479255 = \frac{\Delta P(\ 0.0025)}{2 \times \mu_m \times 0.0347}$$

$$3437.13066730297547479255 = \frac{\Delta P(0.0360230547550432276657)}{\mu_m}$$

$$\Delta P = 95414.747324330599180257 2\mu_m$$

$$\Delta P = 95414.747324330599180257 2(9.545644544159Hct + 0.0015)$$

$$\Delta P = 910795.262228805927341841 Hct + 143.122120986495898770385$$

Table 2. Mathematically modulated blood pressure drop v/s haematocrit

Date	Haematocrit $(3 \times HB)$ (kg/l)	BPD (Blood Pressure drop) Pascal–second
05/09/2024	0.42	382677.1322
16/09/2024	0.339	308902.7160
28/09/2024	0.336	306170.3302
10/10/2024	0.351	319832.2591
26/10/2024	0.369	336226.5738

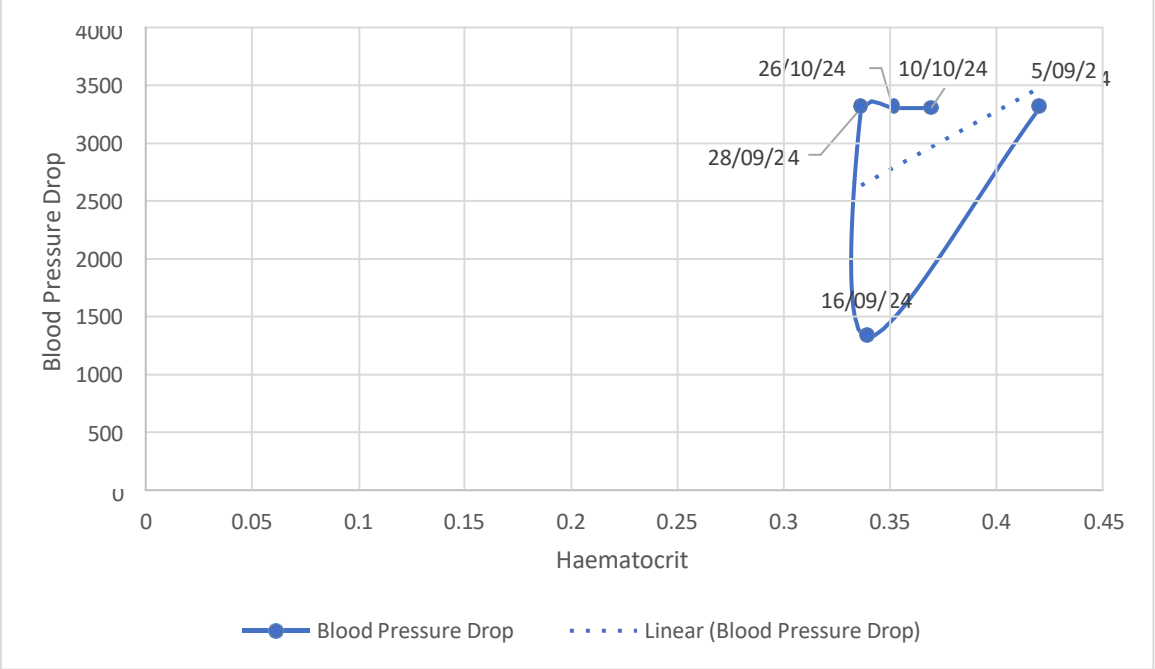

Fig 2. Graphical presentation Haematocrit V/S Blood Pressure Drop (Clinical data)

Observation (a)

Fig.2 (table- I) relation between real clinically haematocrit V/S blood pressure drop,$BPD = (\frac{S+D}{2} - S)$, Where S = Systolic blood pressure and D = Diastolic blood pressure

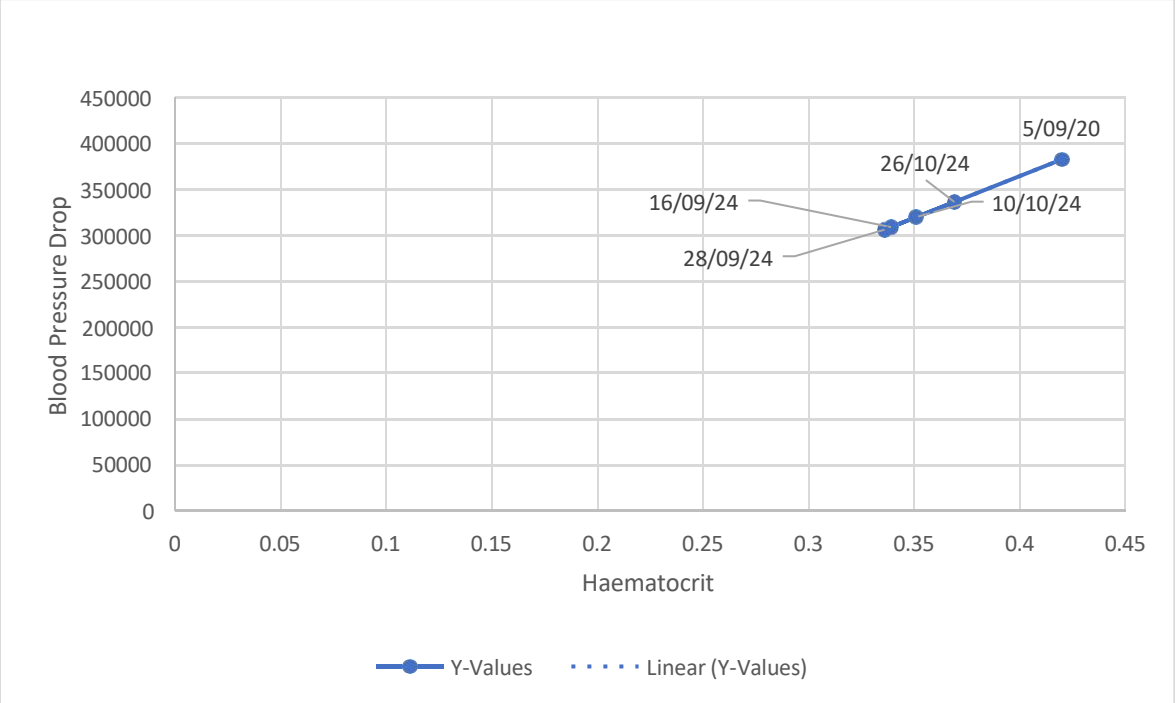

Fig 3. Graphical presentation Haematocrit V/S Blood Pressure Drop (Moderated data)

Observation (b)

Fig 3. (table 2) relations between mathematically modulated haematocrit v/s blood pressure drop.$\Delta P = 900922.85430689016468139 8Hct + 141.570773477758517093084$, Where ΔP is denoted by Relation between blood pressure drop v/s haematocrit (trained line).

CONCLUSION:

In Fig (3) The relation between haematocrit and blood pressure drop. In date 05/09/2024 to 26/10/2024 the graph is showing the decreasing sense date wise. In this situation high dose for the patient is suggested so for.

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REFERENCES

[1] World Health Organization. (2023). Global breast cancer initiative implementation framework: assessing, strengthening and scaling-up of services for the early detection and management of breast cancer. World Health Organization.

[2] Mohanty, S. K., Wadasadawala, T., Sen, S., & Khan, P. K. (2023). Socio-economic variations of breast cancer treatment and discontinuation: a study from a public tertiary cancer hospital in Mumbai, India. *BMC women's health*, *23*(1), 113. https://doi.org/10.1186/s12905-023-02275-6

[3] Lee, C. L., Tsai, C. H., Yeh, D. C., Lin, C. S., Li, Y. F., & Tzeng, H. E. (2017). Hemoglobin level trajectories in the early treatment period are related with survival outcomes in patients with breast cancer. *Oncotarget*, *8*(1), 1569-1579. https://doi.org/10.18632/oncotarget.13679

[4] Mercadante, A. A., & Raja, A. (2019). Anatomy, arteries.

[5] Wolff, C. B., Collier, D. J., Shah, M., Saxena, M., Brier, T. J., Kapil, V., Green, D., & Lobo, M. (2016). A Discussion on the Regulation of Blood Flow and Pressure. *Advances in experimental medicine and biology*, *876*, 129–135. https://doi.org/10.1007/978-1-4939-3023-4_16

[6] Upadhyay, V., Pandey, P., Chaturvedi, S., & Upadhyay, A. (2012). A mathematical model on effect of stenosis in two-phase blood flow in arteries remote from the heart. *International Academy of Physical Sciences (IAPS), 16*, 247-257.

[7] Upadhyay V., (2001), "Some Phenomena in Two-Phase Blood Flow" Ph.D. Thesis, Central University, Allahabad, pp.123.

[8] Guyton, A. C., & Hall, J. E. (2006). Textbook of medical physiology (11th ed.).

[9] Murray, R. S. (2021). *Vector analysis and an introduction to tensor analysis*. Schaum's Outline Series. McGraw-Hill: New York.

[10] Elert, G. (1998–2019). Viscosity. The physics hypertext book. Retrieved from http://www.physics.info/viscosity/

[11] Gustafson, D. R. (2010). Physics, health, and the human body, Wadswor.

[12] Spain, B. (2003). *Tensor calculus: A concise course*. Courier Corporation.

[13] Ram, R., Kumar, D., Paik, P., & Sarkar, A. (2023). A simple and low-cost paper-based device for simultaneous determination of hematocrit and hemoglobin levels in point-of-care settings. AIP Advances, 13(7), 075111. https://doi.org/10.1063/5.0176299.