SEMI-ANALYTICAL SOLUTION FOR HEAT AND MASS TRANSFER ON MHD FLOW OF NANO PARTICLES

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Abstract:

The non-linear differential equations govering the heat and mass transfer on MHD flow of nano particles are solved analytically. The Homotopy Analysis method is adapted to obtain the dimensionless velocity, dimensionless temperature and dimensionless concentration. The results thus obtained are used to obtain the quantitude engineering interest like Nusselt number Nu and Sherwood number Sh. The results are discussed graphically and finds good agreement with the numerical solution.

Keywords:

Non-linear differential equations; Homotopy analysis method; Dimensionless velocity; Dimensionless temperature; Dimensionless concentration; Nusselt number; Sherwood number.

1. INTRODUCTION

These days, there are several real-world scenarios where we must operate with various kinds of nanoparticles. These nanoparticles are essential for regulating the various thermo physical characteristics of various fluids that are involved. The majority of practical fluids, including water, ethylene, glycol, kerosene oil, and engine oil, are poor heat conductors. One of the main causes of lower thermal conductivity and other thermal property values is this. Nanoparticles are added to the base fluids to address this issue and improve the fluids' thermal characteristics. Numerous researchers looked at and suggested a number of models to obtain a tangible study of these nanofluids based on nanoparticles Several models were provided by Choi [1], Choi et al. [2], Buongiorno [3], Nield and Kuznetsov [4], and Kuznetsov and Nield [5] to examine different aspects of researchers used these models to study various problems involving nanofluids that can be seen in Refs. [6–13].

Examining the diffusion of vorticity across an abruptly shifted flat surface is the classical Stokes' problem [14]. Na and Rajagopal [15] expanded on the customary issue in the non-Newtonian fluid scenario. Numerous researchers have examined various velocity field properties in Stokes' issue over the years. The literature on Stokes' problem has numerous research [16–31]. Following the groundbreaking research on nanofluids, Naseem UddinA study of nanofluids caused by a suddenly shifted plate was presented by et al. [32]. Using the Buongiorno model, Rosali et al. [33] investigated the effects of thermophoresis and Brownian motion. on mass transfer and heat in nanofluids.

A fresh, updated model that cooperated with the zero flux boundary requirement for the concentration profile at the wall was offered by Nield and Kuznetsov [34–36] in a recent work. This incorporates thermophoresis and Brownian motion dynamics to provide passive control of the nanofluid particle fraction at the wall. They added that, in comparison to previous models, this one is more physically accurate.

The current study examines the flow behavior of a nanofluid across a quickly shifted flat plate using the model proposed by Nield and Kuznetsov [34–36]. Impacts of The zero flux border condition incorporates thermophoresis and Brownian motion. Once the partial differential equations have been converted to a set of ordinary differential equations, the flow's governing equations are solved using the well-known Runge-Kutta-Fehlberg method. With the help of graphical aids, a thorough explanation of the impacts of emerging parameters is given.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Examine the flow of a Newtonian nanofluid across a hot plate that has been begun impulsively. Coordinates in Cartesian system is taken to describe the flow behavior. x and y respectively are the coordinates along and normal to the plate. Initially (at time t=0), the plate is starts moving with a constant velocityU_∞. The plate is kept at a constant temperature T_W, and the nanoparticle volume fraction Cw. At a large distance from the plate, the temperature and the nanoparticle volume fraction are represented by T_∞ and C_∞, respectively. A uniform time dependent transverse magnetic field is applied in y-direction. Strength of magnetic field is taken to be $B = B_0$ Induced magnetic field is assumed to be very small as compared to the applied magnetic field and is neglected. Figure 1 shows the schematic diagram for the flow problem.

Fluid phase and the nanoparticles are assumed to be in a thermal equilibrium and there is no slip between them. Under the assumptions mentioned above, the equations governing the flow are [18]:

$$\frac{\partial u}{\partial t} = \nu\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma B_0^2}{\rho_f} u, \tag{1}$$

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tau\left[D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{2}$$

$$\frac{\partial C}{\partial t} = D_B\left(\frac{\partial^2 C}{\partial y^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right), \tag{3}$$

where, u is the velocity component along the y-axis, U_∞ is the free stream velocity, ρf is the density of base fluid, $\nu = \frac{\mu_f}{\rho_f}$ is kinematic viscosity, σ is electrical conductivity, B_0 magnetic field flux density, α thermal diffusivity, D_B is Brownian motion diffusion coefficient, D_T thermophoresis diffusion coefficient, T and C are fluid temperature and nanoparticle volume fraction, respectively, τ is the parameter defined by $\frac{(\rho c)_f}{(\rho c)_p}$, where $(\rho c)_f$ is the heat capacity of the nanofluid and $(\rho c)_p$ is the effective heat capacity of the nanoparticle material.

For active control model, the initial and boundary conditions are:

$$t < 0: \quad u = 0, \quad T = T_W, C = C_W, \text{ for all x,y}$$

$$t \geq 0: u = U_\infty, T = T_W, C = C_W \, at y = 0 \tag{4}$$

$$u \to 0, T \to T_\infty, C \to C_\infty, as y \to \infty$$

The initial and boundary conditions for passive control model are:

$$t < 0: \quad u = 0, \quad T = T_W, C = C_W, \text{ for all x,y}$$

$$t \geq 0: u = U_\infty, T = T_W, D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0, at y = 0 \tag{5}$$

$$u \to 0, T \to T_\infty, C \to C_\infty, as y \to \infty$$

It is pertinent to mention that the last part of the boundary condition takes the thermosphoresis into account and normal flux of the nanofluid at the boundary is taken to be zero [21–23]. The major purpose of this condition is the passive control of nanoparticle volume fraction at the boundary. This condition is different from the earlier studies and gives a more realistic approach. For mathematical analysis of the problem for active control model, we use following similarity transformations

$$\eta = \frac{y}{\sqrt{vt}}, \theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty},$$

$$\varphi(\eta) = \frac{C - C_\infty}{C_W - C_\infty}, u = U_\infty F(\eta), \tag{6}$$

While, for passive control model, following transformation for the concentration is used:

$$\varphi(\eta) = \frac{C - C_\infty}{C_\infty}, \tag{7}$$

After the implementation of Eqs (6) and (7) into Eqs. (1)–(4), we get the following system of nonlinear differential equations,

$$F'' + 2\eta F' - 4M^2 F = 0, \tag{8}$$

$$\theta'' + 2\Pr\eta\theta' + \Pr Nb\theta'\varphi' + \Pr Nt\theta'^2 = 0, \tag{9}$$

$$\varphi'' + 2\Pr Le\eta\varphi' + \frac{Nt}{Nb}\theta'' = 0. \tag{10}$$

The boundary conditions (4) reduce to using Eq. (6).

$$F(0) = 1, \theta(0) = 1, \varphi(0) = 1,$$

$$F(\infty) \to 0, \theta(\infty) \to 0, \varphi(\infty) \to 0, \tag{11}$$

Equation (7) transforms the boundary conditions (5) into

$$F(0) = 1, \theta(0) = 1, Nb\varphi'(0) + Nt\theta'(0) = 0,$$

$$F(\infty) \to 0, \theta(\infty) \to 0, \varphi(\infty) \to 0, \tag{12}$$

where the distinction with respect to η is indicated by primes.

Additionally,

$$M=\frac{\sigma v t B_0^{\,2}}{\rho f a},\ \Pr=\frac{v}{\alpha_m},\ Le=\frac{\alpha_m}{D_B},\ Nt=\frac{\tau D_T(T_W-T_\infty)}{T_\infty v}$$

$$Nb=\frac{\tau D_B(C_W-C_\infty)}{v}$$

stand for the Brownian motion parameter, Lewis number, Prandtl number, magnetic number, and thermophoresis parameter, in that order.

The quantities of engineering interest are the local Nusselt number Nu and Sherwood number Sh. These metrics describe the nano mass and wall heat.corresponding transfer rates, and are described by [33]:

$$Nu=-\frac{1}{2}\theta'(0), \tag{13}$$

$$Sh=-\frac{1}{2}\phi'(0). \tag{14}$$

Additionally, the Sherwood number expression becomes 0 for the passive control paradigm. Specifically, Sh =0.

3. SEMI-ANALYTIC EXPRESSION BY HOMOTOPY ANALYSIS METHOD [37]-[47]

The Homotopy analysis method (HAM) is an analytical approximation method for highly non- linear problems, proposed by Liao in 1992. Unlike other perbutation techniques, HAM is independent of any small/large physical parameters. Secondly, HAM provides a convenient way to guarantee the convergence of solution. Finally, HAM contains some traditional methods. Therefore HAM provides us a useful tool to solve highly nonlinear problems in science and engineering [37-47]

Dimensionless velocity, temperature and concentration are given by:

$$f_0=e^{-\eta}$$

$$f_1=Ce^{-\eta}+De^{-2\eta}+(-1+4M^2)\eta e^{-\eta}+2e^{-\eta}\left(\frac{\eta^2}{2}-\eta\right)$$

Where C = -D

$$\theta_0 = e^{-\eta}$$

$$\theta_1 = Ce^{-\eta} + De^{-\eta} - \eta e^{-\eta} + e^{-\eta}\left(\frac{\eta^2}{2} - \eta\right)(2pr) + \eta e^{-2\eta}(prNb + prNt)$$

Where $C = -D$

Active control flow:

$$\varphi_0 = e^{-\eta}$$

$$\varphi_1 = Ce^{-\eta} + De^{-2\eta} + \eta e^{-\eta}\left[-1 + \frac{Nt}{Nb}\right] + e^{-\eta}\left(\frac{\eta^2}{2} - \eta\right)(2prLe)$$

where C= -D

Passive control flow:

$$\varphi_0 = 2e^{-\eta} - \left(1 + \frac{N_t}{2Nb}\right)e^{-2\eta}$$

$$\varphi_1 = Ce^{-\eta} + De^{-2\eta} - \left(2 + \frac{Nt}{Nb}\right)e^{-\eta} - 4\left(1 + \frac{Nt}{2Nb}\right)\eta e^{-2\eta} + 4\Pr Le\left(\frac{\eta^2}{2} - \eta\right)e^{-\eta}$$

4. RESULT AND DISCUSSION

This section, is dedicated to the study of velocity, temperature and concentration profiles for the varying values of involved parameters. To serve this purpose Figure 1-9 are plotted. Fig 1 describes the behavior of dimensionless velocity profile for increasing values of magnetic number M. It can be see that the increase in M decreases the dimensionless velocity. For both, numerical solution and analytical solution , the dimensionless velocity profile is identical. Fig 2 describes the dimensionless temperature for increasing values of thermophoresis parameter Nt. It can be see that the increase in Nt increase the dimensionless temperature. Fig 3 describes the dimensionless temperature for increasing values of Brownian motion parameter Nb. It can be see that the increase in Nb increase the dimensionless temperature. Effects of thermophoresis parameter Nt, Brownian motion parameter Nb and Lewis number Le on the dimensionless concentration profile are shown in Fig 4-9, respectively. Fig 4&7(active &passive flow) describes the dimensionless concentration for thermophoresis parameter Nt. It can be see that increase in Nt decrease the dimensionless concentration. Fig 5&8(active &passive flow) describes the dimensionless concentration for increasing values of Brownian motion parameter Nb. Fig 6&9(active &passive flow) describes the dimensionless concentration for Lewis number Le.

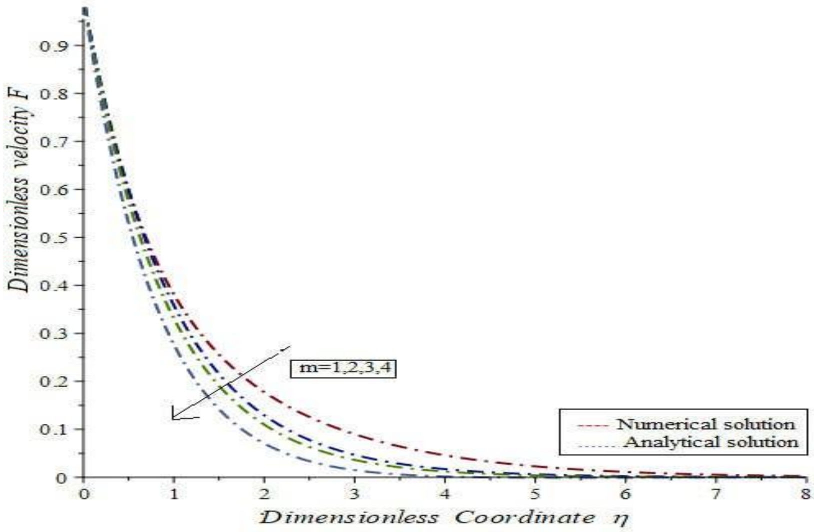

Fig 1: Variation in dimensionless velocity with increasing values of magnetic number M.

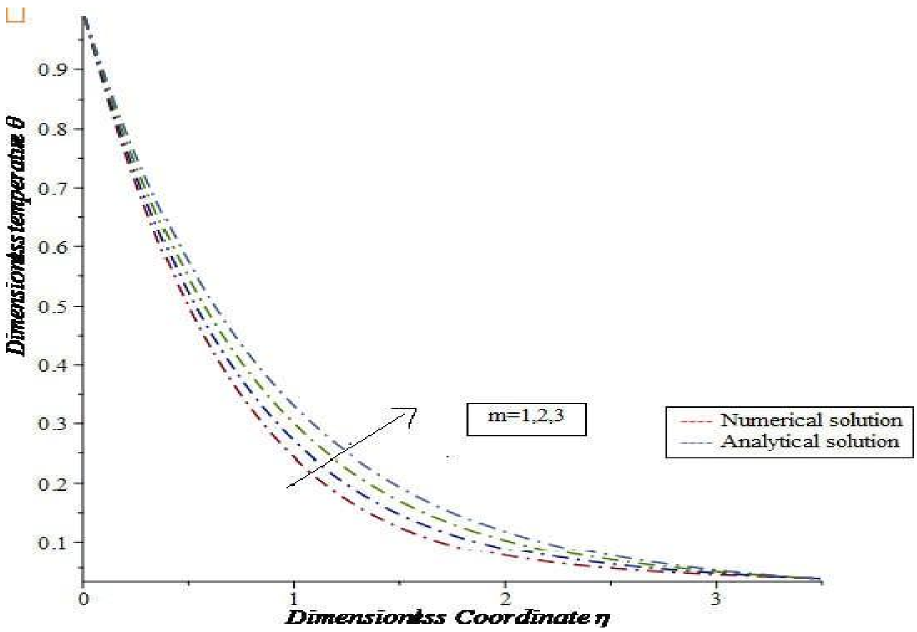

Fig 2: Variation in dimensionless temperature with increasing values of thermophoresis parameter Nt.

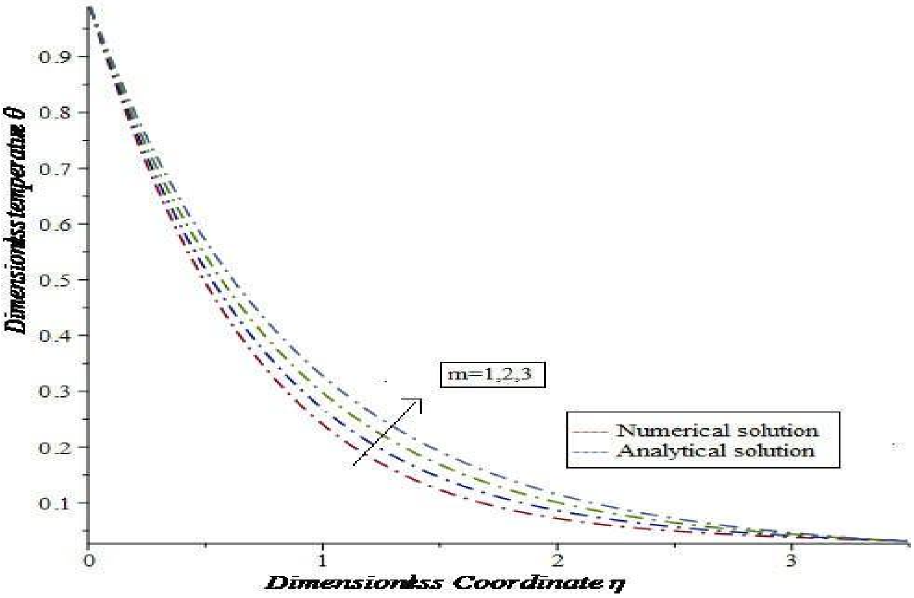

Fig 3: Variation in dimensionless temperature with increasing values of Brownian motion parameter Nb.

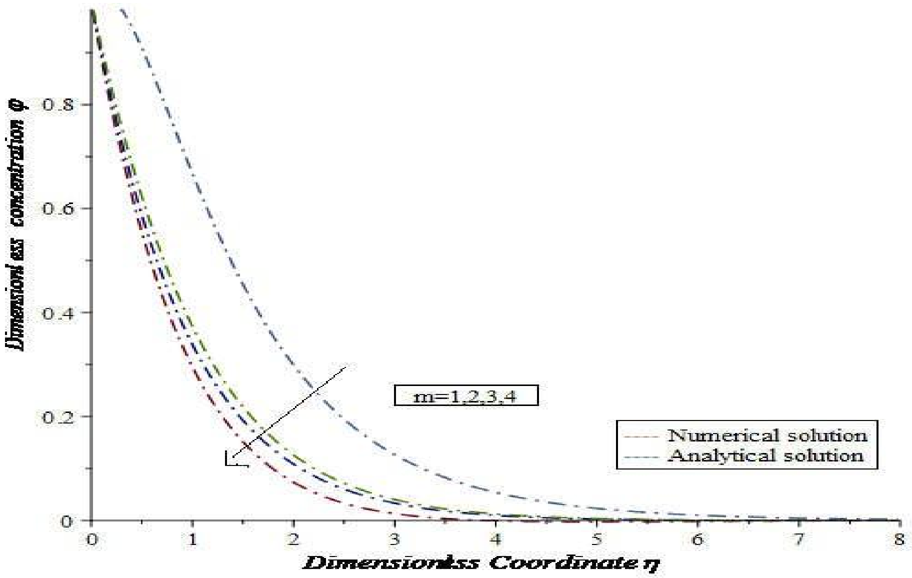

Fig 4: Variation in dimensionless concentration (Active flow) with increasing values of thermophoresis parameter Nt.

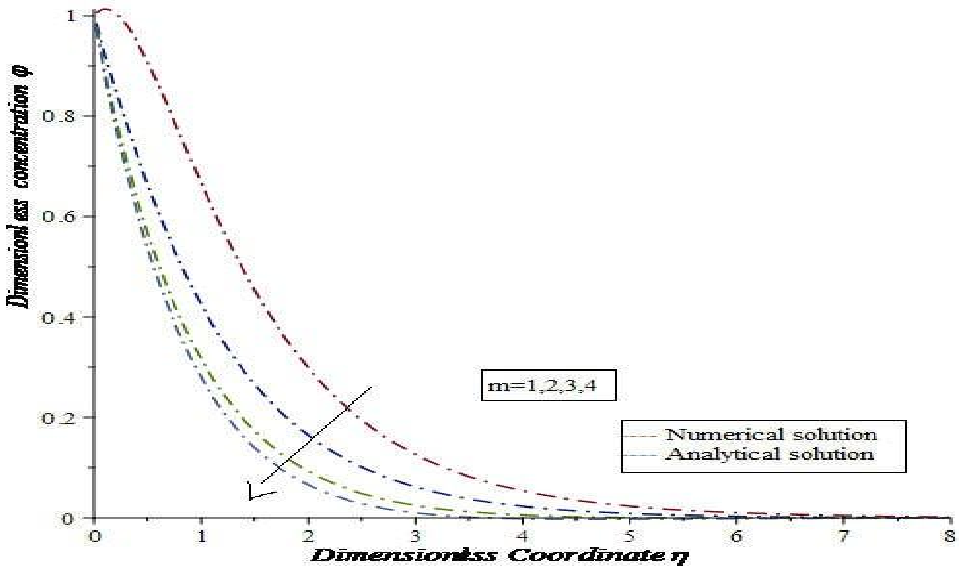

Fig 5: Variation in dimensionless concentration (Active flow) with increasing values of Brownian motion parameter Nb.

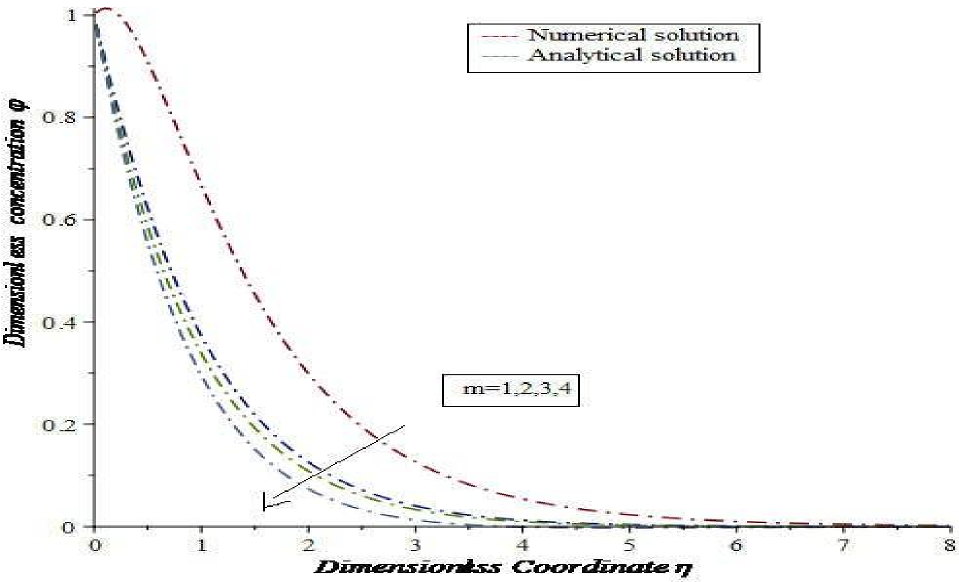

Fig 6: Variation in dimensionless concentration (Active flow) with increasing values of Lewis number Le.

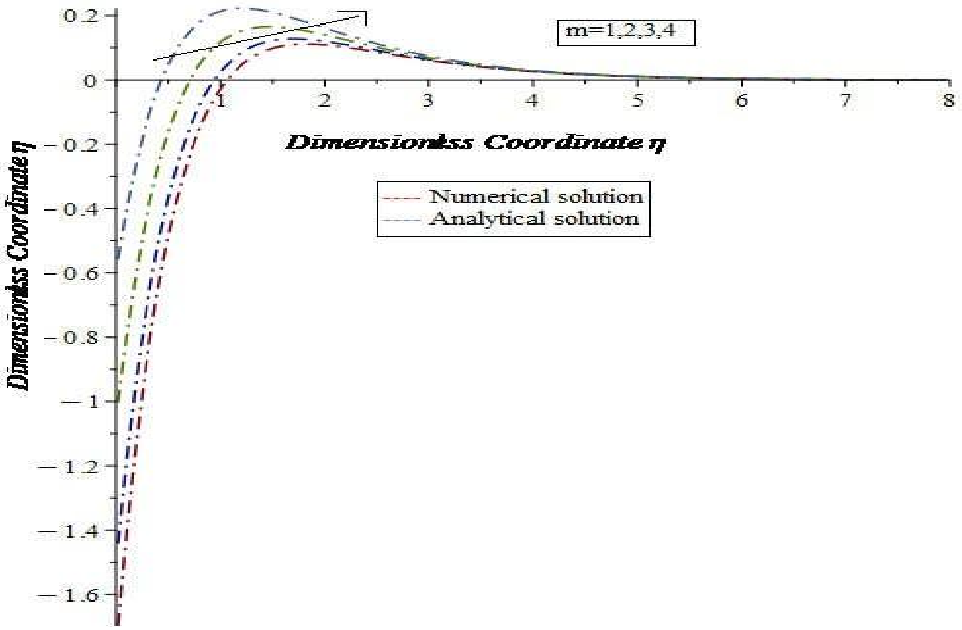

Fig 7: Variation in dimensionless concentration (Passive flow) with increasing values of thermophoresis parameter Nt

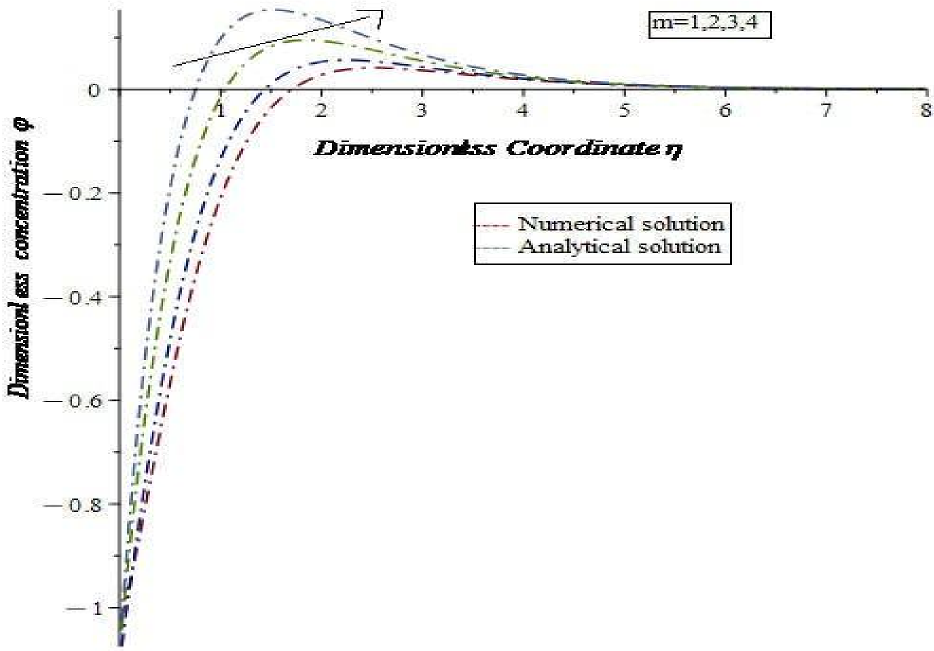

Fig 8: Variation in dimensionless concentration (Passive flow) with increasing values of Brownian motion parameter Nb

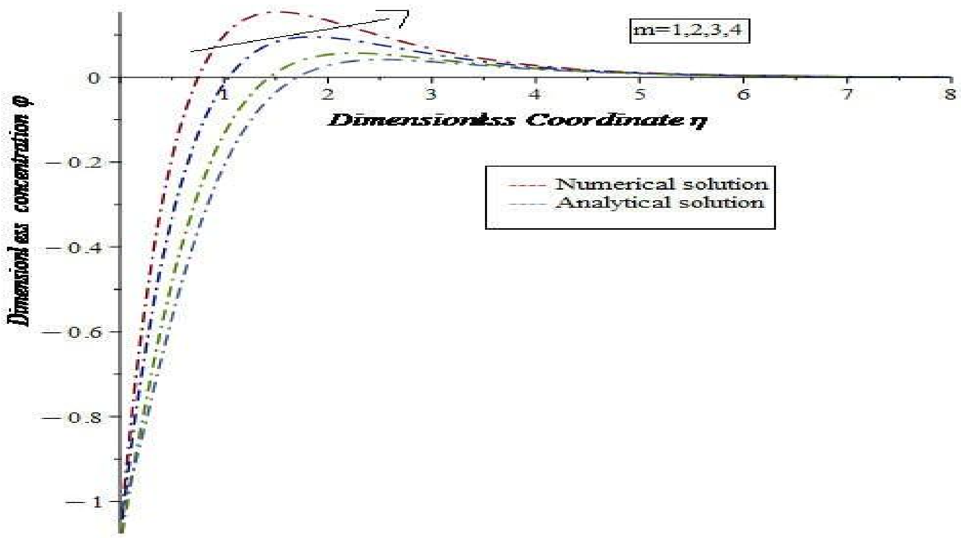

Fig 9: Variation in dimensionless concentration (Passive flow) with increasing values of Lewis number Le.

5. CONCLUSION

In this paper Homotopy analysis method is adapted to study the heat and mass transfer on MHD flow of nanoparticles . Analytical expression for velocity, temperature and concentration is obtained and their impacts on varying the parameter are discussed graphically.

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Nomencalture:

M	Magnetic number
Pr	Prandtl number
Le	Lewis number
Nt	Thermophoresis parameter
Nb	Brownian motion parameter
Nu	Nusselt number
Sh	Sherwood number
η	Similarity parameter

Appendix A: Semi-Analytic expression for dimensionless velocity using HAM:

The non-linear differential equation is

$$F'' + 2\eta F' - 4M^2 F = 0 \qquad (A1)$$

The boundary conditions are

$$F(0) = 1,\ F(\infty) \to 0 \qquad (A2)$$

We construct the Homotopy for the eqn.(A1) as follows:

$$(1-p)\left(f'' + 3f' + 2f\right) + hp\left(f'' + 2\eta f' - 4M^2 f\right) = 0 \qquad (A3)$$

The analytical solution of the eqns.(A1) and (A2) is given by

$$(f_0'' + Pf_1'' + P^2 f_2'' + \ldots\ldots) + 3(f_0' + Pf_1' + \ldots\ldots\ldots) + 2(f_0 + Pf_1 + P^2 f_2 + \ldots.) \qquad (A4)$$

Substituting the eqns. (A4) in the eqn. (A1) and comparing the coefficients of the power of p

We get

$$P^0 :\ f_0'' + 3f_0' + 2f_0 = 0 \qquad (A5)$$

P^1 :

$$f_1'' + 3f_1' + 2f_1 - f_0'' - 3f_0' - 2f_0 + h(f_0'' + 2\eta f_0' - 4M^2 f_0) = 0 \qquad (A6)$$

Solving the eqns. (A5), (A6) and using initial approximation eqns we obtain the following results:

$$f_0 = C_1 e^{-\eta} + C_2 e^{-2\eta}$$

$$f_1 = Ce^{-\eta} + De^{-2\eta} + (-1 + 4M^2)\eta e^{-\eta} + 2e^{-\eta}\left(\frac{\eta^2}{2} - \eta\right)$$

Using the boundary conditions, C= -D

Appendix B: Semi-Analytic expression for dimensionless temperature using HAM:

The non-linear differential equation is

$$\theta'' + 2\Pr\eta\theta' + \Pr Nb\theta'\varphi' + \Pr Nt\theta'^2 = 0, \qquad (B1)$$

The boundary conditions are

$$\theta(0)=1,\ \phi(0)=1$$
$$\theta(\infty)\rightarrow 0,\ \phi(\infty)\rightarrow 0 \qquad (B2)$$

We construct the Homotopy for the eqn. (B1) as follows:

$$(1-p)\left(\theta''+3\theta'+2\theta\right)+hp\left(\theta''+2\Pr\eta\theta'+\Pr Nb\theta'\varphi'+\Pr Nt\theta'^{2}\right)=0 \qquad (B3)$$

The analytical solution of the eqns. (B1) and (B2) is given by

$$(\theta_0''+P\theta_1''+P^2\theta_2''+......)+3(\theta_0'+P\theta_1'+........)+2(\theta_0+P\theta_1+P^2\theta_2+....) \qquad (B4)$$

Substituting the eqns. (B4) in the eqn. (B1) and comparing the coefficients of the power of p

We get

$$P^0;\ \theta_0''+3\theta_0'+2\theta_0=0 \qquad (B5)$$

$$P^1;$$
$$\theta_1''+3\theta_1'+2\theta_1-\theta_0''-3\theta_0'-2\theta_0+\theta_0''+2\Pr\eta\theta_0'+\Pr Nb\theta_0'\varphi'+\Pr Nt\theta_0'^{2}=0 \qquad (B6)$$

Solving the eqns. (B5), (B6) and using initial approximation eqns we obtain the following results:

$$\theta^0=e^{-\eta}$$
$$\theta_1=Ce^{-\eta}+De^{-\eta}-\eta e^{-\eta}+e^{-\eta}\left(\frac{\eta^2}{2}-\eta\right)(2pr)+\eta e^{-2\eta}(prNb+prNt)$$

Using the boundary conditions we get, C= -D

Appendix C: Semi-Analytic expression for dimensionless concentration using HAM:

The non-linear differential equation is

$$\varphi''+2\Pr Le\eta\varphi'+\frac{Nt}{Nb}\theta''=0. \qquad (C1)$$

The boundary conditions are

$$\theta(0)=1,\ \varphi(0)=1$$
$$\theta(\infty)\rightarrow 0,\ \varphi(\infty)\rightarrow 0 \qquad (C2)$$

We construct the Homotopy for the eqn. (C1) as follows:

$$(1-p)\left(\varphi'' + 3\varphi' + 2\varphi\right) + hp\left(\varphi'' + 2\Pr le\eta\varphi' + \frac{Nt}{Nb}\theta''\right) = 0 \quad \text{(C3)}$$

The analytical solution of the eqns. (C1) and (C2) is given by

$$(\varphi_0'' + P\varphi_1'' + P^2\varphi_2'' + ...) + 3(\varphi_0' + P\varphi_1' + ...) + 2(\varphi_0 + P\varphi_1 + P^2\varphi_2 + ...) \quad \text{(C4)}$$

Substituting the eqns. (C4) in the eqn. (C1) and comparing the coefficients of the power of p

We get

$$P^0; \quad \varphi_0'' + 3\varphi_0' + 2\varphi_0 = 0 \quad \text{(C5)}$$

$$P^1;$$

$$\varphi_1'' + 3\varphi_1' + 2\varphi_1 - \varphi_0'' - 3\varphi_0' - 2\varphi_0 + \varphi_0'' + 2\Pr le\eta\varphi_0' + \frac{Nt}{Nb}\theta'' = 0 \quad \text{(C6)}$$

Solving the eqns. (C5), (C6) and using initial approximation eqns we obtain the following results:

$$\varphi_0 = 2e^{-\eta} - \left(1 + \frac{N_t}{2Nb}\right)e^{-2\eta}$$

$$\varphi_1 = Ce^{-\eta} + De^{-2\eta} - \left(2 + \frac{Nt}{Nb}\right)e^{-\eta} - 4\left(1 + \frac{Nt}{2Nb}\right)\eta e^{-2\eta} + 4\Pr Le\left(\frac{\eta^2}{2} - \eta\right)e^{-\eta}$$

Using the boundary conditions, we get C= -D