Analysis of Modified Holographic Ricci Dark energy model in $f(R,T)$ gravity

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Abstract: *We discuss the cosmological analysis of the Bianchi type V space time along with the $f(R,T)$ gravity in the sight of energy momentum tensor and Modified Holographic Ricci Dark Energy (MHRDE). To determine dynamical parameter, we apply the equation of state for the pressure (p_Λ) and density (ρ_Λ) by the relation $\omega_\Lambda = {p_\Lambda}/{\rho_\Lambda}$. We examine the physical and dimensional implications of the obtained model.*

Keywords: $f(R,T)$ gravity, Bianchi type V metric, Modified Holographic Ricci Dark Energy, Matter energy density.

1. Introduction

It is acclaimed by the astronomical observations that our universe is under the influence of dark energy that causes increasing rate of extension of the universe [1,2,3].The more skilled contender of dark energy is cosmological constant that assured the cosmological observations[4,5] but unsuccessful in resolving the problem of fine tuning , cosmic coincidence[6].As Einstein's theory was outstanding for explaining the structure and origin of the universe and also for constructing different cosmological models but unfortunate while explaining the late time acceleration .So in order to modify this theory, some changes must be done. For that, standard Einstein-Hilbert action is restored by $f(R)$ gravity (R is Ricci Scalar) [7]. In [8] the detailed analysis on $f(R)$gravity has been done. Moderation of standard general relativity (GR)is $f(R,T)$gravity, in this Gravitational Lagrangian is inclined by Ricci scalar R with stress of energy momentum tensor T [9]. "Bianchi type V universe model with magnetized domain walls in $f(R,T)$theory" have been investigated in [10]. By taking help of different Bianchi type space times [11,12,13,14,15,16,17] have prepared cosmological models in $f(R,T)$gravity. Basically, the cosmological constant (λ) or dark energy (DE) obstacle is concern with quantum gravity. Since the cosmological constant unavoidably relates to the vacuum expectation value of quantum fields. Doubtlessly, the holographic principle forms a necessary base for the quantum gravity theory. Thus, the holographic principle provides the information about the nature of "cosmological constant" (or dark energy).

In the recent exploration, an attractive effort to check out the description of universe with the connection of quantum gravity is "Holographic Dark Energy" model [18,19,20,21]. The quantum action of black holes is broadly used the holographic principle. In accordance with holographic principle, In a bounded system the number of degrees of freedom must be restricted and is concern with the area of its boundary and this is examined by [22].Certain features of holographic dark energy have been discussed in [23].The holographic dark energy (HDE) density (ρ) is clearly expressed as $\rho = 3c^2 M_{pl}^2 L^{-2}$,Here c is constant.$M_{pl}$represents plank mass which is ${1}/{\sqrt{8\pi G}}$. L is meant to be the size of current universe. Later on, Gao et al [24] put forward a model in which future event horizon is displaced by the inverse of the "Ricci scalar curvature" as $L \approx |R|^{-\frac{1}{2}}$ which is known as "Ricci dark energy model" (RDE). Then Granda and Oliveros [25,26] investigated latest "holographic Ricci dark energy model" by taking energy density in following form,

$$\rho_\Lambda = \frac{3}{8\pi G}(\xi H^2 + \eta \dot{H}),$$

After doing some modifications, Chen and Jing [27] labelled it "Modified Holographic Ricci Dark Energy" (MHRDE) having energy density in the form,

$$\rho_\Lambda = \frac{3}{8\pi G}(\xi H^2 + \eta \dot{H} + \varsigma \ddot{H} H^{-1}),$$

Where ξ,η,ς are three arbitrary parameters, overhead dot shows differentiation with respect to t also, G represents gravitational constant claimed as function of time. Further, several authors have proposed HDE and MHRDE models in their literature in the context of Bianchi space times. Sireesha and Rao [28] proposed "spatially homogeneous anisotropic Bianchi type II, VIII and IX cosmological models in the framework of $f(R,T)$ gravity". Discussion on the "Bianchi type V MHRDE model in Saez-Ballester scalar tensor theory of gravitation" have been done in [29]. Naidu et al. [30] have acquired dynamics of axially symmetric anisotropic MHRDE model in Brans-Dicke theory of gravitation. Pawar et al. [31] discussed "A modified holographic Ricci dark energy model in $f(R,T)$ theory of gravity" in recent times.

Inspired by the exceeding discussion, we have considered the general class of Bianchi type V space time with Modified Holographic Ricci Dark Energy in $f(R,T)$ theory of gravity. Some physical and dimensional features of the obtained model which are thought to be appropriate with current observations.

The major objective of the current work is to analyse Modified Holographic Ricci Dark Energy (MHRDE) model in the framework of $f(R,T)$ gravity. Description of gravity model has been given in Sect. II. In Sect. III we investigate the metric and field equations of the models. Solutions of these field equations are discussed in Sect. IV. Sect. V is concerned with the discussion of the results with its graphical representation. The last Sect.VI contains conclusion of the work.

2. Explanation of $f(R,T)$gravity

We precisely describe $f(R,T)$ gravity which is a generalization or modification of Einstein's theory of relativity. Hilbert-Einstein action for this theory is stated as

$$S = \int L_m \sqrt{-g}d^4x + \frac{1}{2k}\int f(R,T)\sqrt{-g}d^4x \tag{1}$$

here $f(R,T)$ is an arbitrary function of R (Ricci Scalar) and T (trace of energy momentum tensor). By differing the action (1) of the gravitational field regarding to the metric tensor g_{ij} we got the field equations for $f(R,T)$ gravity by

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} - \left(\Delta_i\Delta_j - g_{ij}\Box\right)f_R(R,T) = kT_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \tag{2}$$

here ∇_i is the derivative along tangent vector, $f_R(R,T) = \frac{\partial f_R(R,T)}{\partial R}$, $f_T(R,T) = \frac{\partial f_R(R,T)}{\partial T}$, $\Box = \nabla^i\nabla_i$,

$\theta_{ij} = g^{lk}\frac{\delta T_{lk}}{\delta g^{ij}}.k = \frac{8\pi G}{c^4}$ here c represents the speed at which light travels in vacuum, G represents Newtonian Gravitational constant.

$$T_{ij} = T'_{ij} + \bar{T}_{ij} \quad (3)$$

here T_{ij} represents the standard energy momentum tensor obtain from matter Lagrangian L_m in which T'_{ij} serves as energy momentum tensor and $\bar{T}_{ij}$ represents the energy momentum tensor in order to modified holographic Ricci dark energy.

$$T'_{ij} = \rho_M u_i u_j \, , \ i,j = 1,2,3,4 \quad (4)$$

$$\bar{T}_{ij} = (\rho_\Lambda + p_\Lambda)u_i u_j + g_{ij}p_\Lambda \quad (5)$$

The four-velocity vector in co-varying coordinates $u^i = (0,0,0,1)$ satisfies the condition

$u^i\nabla_j u_i = 0$ and $u^i u_i = 1$. In this current work, we have chosen $f(R,T)$in a particular way as $f(R,T) = R + 2f(T)$, here $f(T)$ is random function of T. So, the analogues field equations are expressed by

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = kT_{ij} + 2f_T T_{ij} + [f(T) + 2p_\Lambda f_T]g_{ij} \quad (6)$$

where f_T is the first derivative of f respect to T. Let $f(T) = \lambda T$, here λ is constant. From Equation (5), we have

$$\bar{T}^i_j = diag[-1, \omega_x, \omega_y, \omega_z]\rho_\Lambda = diag[-1, \omega_\Lambda, (\omega_\Lambda + \delta), (\omega_\Lambda + \gamma)]\rho_\Lambda \quad (7)$$

here $\omega_x, \omega_y, \omega_z$ represents the directional equation of state parameter with x, y, z axes respectively,

The equation of state parameter is taken by

$$\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} \quad (8)$$

here we take $\omega_\Lambda = 1$.

3. Metric and Field equations

The general class of Bianchi type V space time is stated by the metric as,

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}[B^2(t)dy^2 + C^2(t)dz^2] \quad (9)$$

here A, B and C are the functions of cosmic time t , m is arbitrary constant.

By making use of co-moving coordinate system, the equation (6) with equation (4) and equation (7) for the metric (9), credibly described by

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = p_\Lambda + \lambda(8p_\Lambda + \rho_M) \quad (10)$$

$$\frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{c}}{c} + \frac{\ddot{C}}{C} - \frac{m^2}{A^2} = p_\Lambda + \lambda(8p_\Lambda + \rho_M) \quad (11)$$

$$\frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\ddot{B}}{B} - \frac{m^2}{A^2} = p_\Lambda + \lambda(8p_\Lambda + \rho_M) \quad (12)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{C}\dot{B}}{CB}-\frac{3m^2}{A^2}= p_\Lambda+\rho_M+\lambda(6p_\Lambda+3\rho_M+2\rho_\Lambda) \tag{13}$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 \tag{14}$$

An overhead dot is the derivative respecting to cosmic time t .

After taking integration of (14) and consuming the integral constant, we have

$$A^2=BC \tag{15}$$

For solving the field equations and for analysing the solution we are proceedings towards the dynamical parameters.

The average scale factor $a(t)$ and the spatial volume (V) are clearly expressed by

$$a(t)=(ABC)^{\frac{1}{3}} \tag{16}$$

$$V=a^3(t)=ABC \tag{17}$$

The average Hubble parameter (H) is expressed as

$$H=\frac{1}{3}(H_1+H_2+H_3) \tag{18}$$

Here $H_1=\frac{\dot{A}}{A}$, $H_2=\frac{\dot{B}}{B}$, $H_3=\frac{\dot{C}}{C}$ represents the directing Hubble parameter directed towards x, y, z axes respectively.

The expansion scalar θ , shear scalar σ^2 are describe by

$$\theta=3H=3\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \tag{19}$$

$$\sigma^2=\frac{1}{2}\sigma^{ij}\sigma_{ij} \tag{20}$$

The mean anisotropic parameter Δ is defined by

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{21}$$

The deceleration parameter q is given by

$$q=-1+\frac{d}{dt}\left(\frac{1}{H}\right) \tag{22}$$

4. Solution of field equation

In this segment, we perceive the solution of field equations (10)-(14) in the form as

$$\frac{\dot{A}}{A}=\frac{\dot{B}}{B}=\frac{\dot{C}}{C}=l \tag{23}$$

where l is a (positive) constant.

On integrating equation (23) we have,

$$A(t)=B(t)=C(t)=e^{lt+k} \tag{24}$$

where k is constant of integration. Using equation (24), the metric equation (9) takes the form as,

$$ds^2 = dt^2 - e^{2(lt+K)}[dx^2 + e^{2m} (dy^2 + dz^2)] \quad (25)$$

5. Discussion of Dynamical parameters

Equation (25) represents the General class of Bianchi type-V metric in $f(R,T)$ gravity along with the following dynamical parameters which play a major role at the conversation of cosmology.

The average scale factor $a(t)$ determined to be

$$a(t) = e^{(lt+K)} \quad (26)$$

The spatial volume V is

$$V = e^{3(lt+K)} \quad (27)$$

The Hubble parameter H is

$$H = l \quad (28)$$

The Scalar expansion θ is

$$\theta = 3H = 3l \quad (29)$$

The Shear scalar is

$$\sigma^2 = \frac{3}{2}l^2 \quad (30)$$

The mean anisotropy parameter gives value

$$\Delta = 0 \quad (31)$$

The deceleration parameter q is

$$q = -1 \quad (32)$$

Now from Equation (8), (10), (11) and (24) the matter energy density is determined as

$$\rho_M = \frac{-2}{(1+2\lambda)}\left[\frac{m^2}{e^{2(lt+K)}}\right], \ \lambda \neq \frac{-1}{2} \quad (33)$$

Also, from Equation (8), (10), (11), (24) and (32) the modified holographic Ricci dark energy pressure and density are evaluated by following,

$$\rho_\Lambda = p_\Lambda = \frac{1}{8\lambda+1}\left[3l^2 - \frac{m^2}{e^{2(lt+k)}(1+2\lambda)}\right], \ \lambda \neq \frac{-1}{8} \quad (34)$$

Above results are beneficial while discussing the behaviour of current model (25). We noticed the following observations.

(I)The model (25) gives an expanding, shearing, non-rotating and accelerating universe.

(II) At $t = 0$, the metric potential $A(t)$ and $B(t)$ are constant. Hence this model is free from initial singularity.

(III) The Deceleration parameter (DP)gives $q = -1$ and scalar expansion θ gives constant value that means universe is accelerating with constant rate of expansion.

(IV) The average scale factor increases as cosmic time increases.

(V) At initial epoch, the spatial volume (V) is finite and it increases with increasing cosmic time.

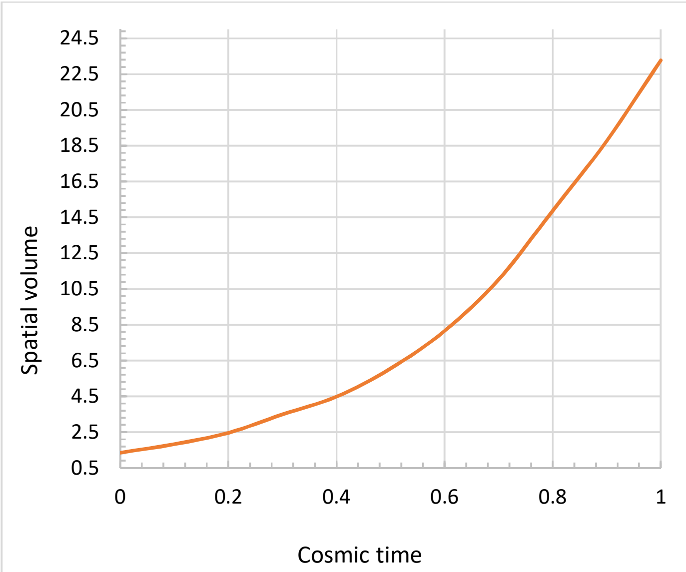

Figure 1. Graph of spatial volume against cosmic time t for $l = 1, k = 0.1$, here t and v are in arbitrary units.

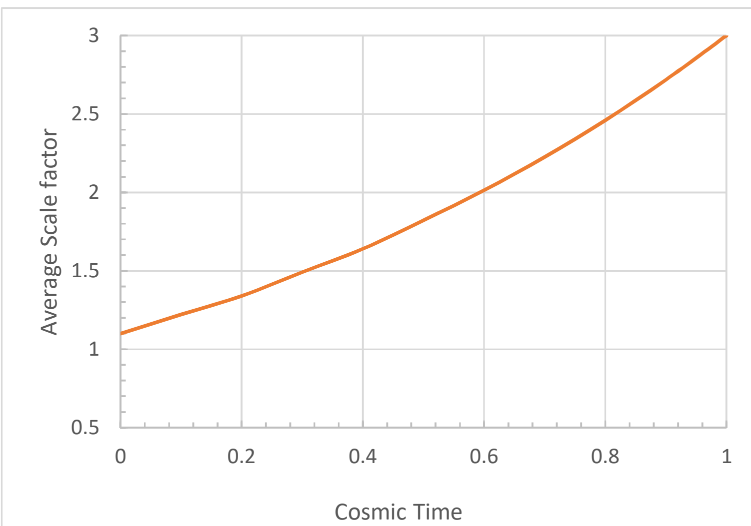

Figure 2. Average scale factor versus cosmic time t for $l = 1, k = 0.1$, here t and a(t) are in arbitrary units.

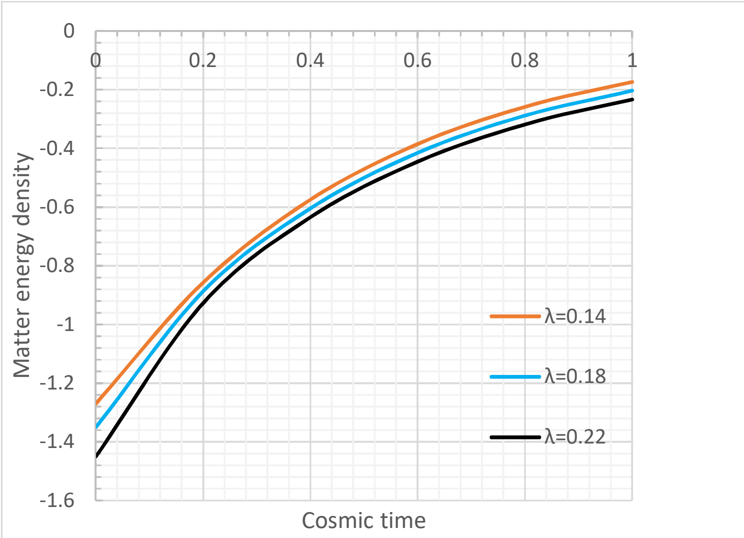

Figure 3. Graph of Matter energy density versus cosmic time t for $l = 1, k = 0.1, m = 1$ with variable constant $\lambda = 0.14, 0.18, 0.22$, here t and ρ_M are in arbitrary units.

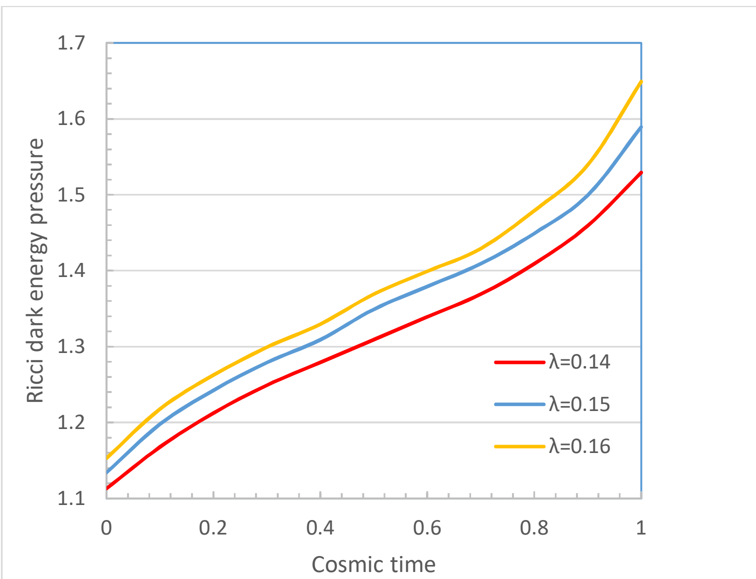

Figure 4. Ricci dark energy pressure versus cosmic time t for $l = 1, k = 0.1, m = 1$ by taking Variable constant $\lambda = 0.14, 0.15, 0.16$, here t and p_Λ are in arbitrary units.

From Figure 1, We observe that the spatial volume extended with increased cosmic time.

Figure 2 confirms that as cosmic time increases as the average scale factor increase.

Figure 3 confirms that ρ_M approaches to zero for greater value of cosmic time.

Figure 4 shows that modified Ricci dark energy pressure increases for increased cosmic time.

6. Conclusion

In this current investigation, study of modified holographic Ricci dark energy (MHRDE) model with the help of general class of Bianchi type -V space time in the presence of $f(R,T)$ gravity has been done. The field equations of the proposed theory have been disclosed by using relation between metric potentials A and B. To determine dynamical parameter, we apply the equation of state for the pressure (p_Λ) and density (ρ_Λ) by the relation $\omega_\Lambda = {p_\Lambda}/{\rho_\Lambda}$. Obtained model is free from physical singularity, it confesses that the expansion of universe starts with finite volume. The average Hubble parameter H as well as the expansion scalar θ are constant, it means Universe is accelerating with constant rate of expansion. We found that Ricci dark energy pressure (p_Λ) increases as time increases. The negative value of deceleration parameter q indicates accelerating Universe. Also, as cosmic time t increases matter energy density(ρ_M) approaches to zero. The anisotropic parameter (Δ) gives its value zero it means that our model proceeds towards isotropy.

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