MAXIMAL INCIDENCE EDGE PRIME GRAPHS

Dr. Sunoj B S
Associate Professor, Department of Mathematics , Government Polytechnic College, Attingal

ABSTRACT . In this paper I introduce a new labeling. I prove that some tree graphs are maximal incidence edge prime graphs. Further I prove sum of the edge values is equal to sum of the vertex values.

1. INTRODUCTION

I consider simple finite and undirected graphs. For all terminology and notations I follow Harary[2].First I provide some definitions useful for present work.

Definition 1.1 Star graph is a special type of graph in which n vertices have degree one and a single vertex have degree n. This graph is denoted by the symbol $K_{1,n}$.

Definition 1.2 Graph obtained by adding single pendant edge to each vertex of a path P_n is called comb graph and is denoted by $P_n \Theta K_1$.

Definition 1.3 Graph obtained by adding two pendant edges to each vertex of path P_n is called centipede graph and is denoted by $P_n \Theta K_{1,2}$.

Definition 1.4 Graph obtained by adding two pendant edges to each internal vertex of path P_n is called twig graph and is denoted by $T_w(n)$.

Definition 1.5 Bi star B(n,n) is the graph obtained by joining the apex vertices of two copies of $K_{1,n}$ by an edge.

Definition 1.6 A Coconut Tree CT(m, n) is the graph obtained from the path Pn by appending m new pendent edges at an end vertex of P_n .

Definition 1.7 A graph obtained from a given graph by breaking up each edge into two edges by inserting a vertex between its two ends is called sub division graph.

Definition 1.8 Greatest common incidence number (gcin) of a vertex of degree two or more is the greatest common divisor of the labels of the edges incident on that vertex.

Definition 1.9 A labeled graph is said to be an incidence edge prime graph, if the gcin of all vertices of degree greater than one is one.

Definition 1.10 A labeled graph is said to be a maximal incidence edge prime graph, if the edge label is the maximum of the end vertex labels.

2. MAIN RESULTS

Theorem 2.1 Path P_n is a maximal incidence edge prime graph.

Proof: Let $a_1,a_2,\ldots,a_n$ be the vertices of the path. Then $|V(P_n)| = n$ and $|E(P_n)| = n-1$.

I define a function $f : V \rightarrow \{0,1,\ldots,n-1\}$ which labels the vertices by

$$f(a_i) = i-1 \text{ for all i from 1 to n.}$$

I define a function $f^*: E \rightarrow \{1,2,\ldots,n-1\}$ which labels the edges by

$$f^*(a_i a_{i+1}) = \text{Max of } \{f(a_i), f(a_{i+1})\}$$

$= \text{Max of } \{i-1, i\}$
$= i$, for all i from 1 to n-1.

gcin of $(a_{i+1}) = \text{gcd of } \{f^*(a_i a_{i+1}), f^*(a_{i+1} a_{i+2})$
$= \text{gcd of } \{i, i+1\}$
$= 1$, for all i from 1 to n-2.

Hence path P_n is a maximal incidence edge prime graph.

Example 2.1 Maximal incidence edge prime labeling of path P_5 is shown in Figure 1.

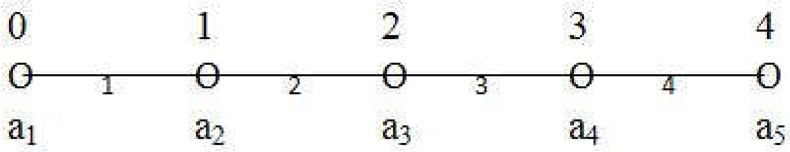

Figure 1

Theorem 2.2 Star graph $K_{1,n}$ is a maximal incidence edge prime graph.

Proof: Let a, $a_1, a_2, \ldots, a_n$ be the vertices of the graph. Then $|V(K_{1,n})| = n+1$ and $|E(K_{1,n})| = n$.

I define a function $f : V \rightarrow \{0,1,\ldots,n\}$ which labels the vertices by

$f(a_i) = i$ for all i from 1 to n.
$f(a) = 0$.

I define a function $f^*: E \rightarrow \{1,2,\ldots,n\}$ which labels the edges by

$f^*(aa_i) = \text{Max of } \{f(a), f(a_i)\}$
$= \text{Max of } \{0, i\}$
$= i$ for all i from 1 to n.

gcin of (a) $= \text{gcd of } \{f^*(aa_1), f^*(aa_2)$
$= \text{gcd of } \{1, 2\}$
$= 1$.

Hence $K_{1,n}$ is a maximal incidence edge prime graph.

Example 2.2 Maximal incidence edge prime labeling of $K_{1,4}$ is shown in Figure 2.

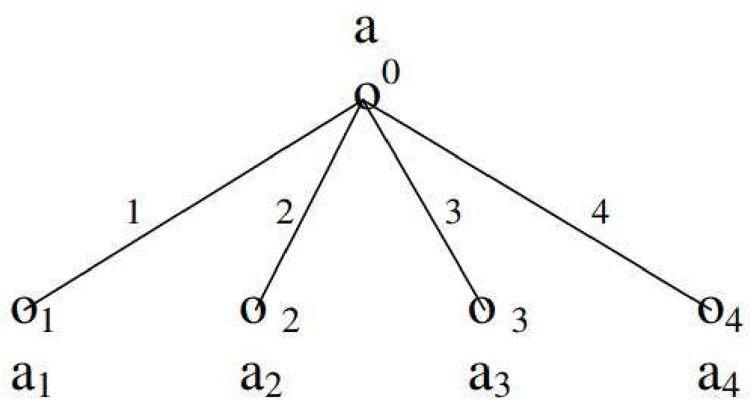

Figure 2

Theorem 2.3 Comb graph $P_n \odot K_1$ is a maximal incidence edge prime graph.

Proof: Let $a_1, a_2, \ldots, a_n$, $b_1, b_2, \ldots, b_n$ be the vertices of the graph. Then $|V(P_n \odot K_1)| = 2n$ and $|E(P_n \odot K_1)| = 2n-1$.

I define a function $f : V \rightarrow \{0,1,\ldots,2n-1\}$ which labels the vertices by

$f(a_i) = 2i-2$ for all i from 1 to n.

$f(b_i) = 2i-1$ for all i from 1 to n.

I define a function $f^*: E \rightarrow \{1,2,\ldots,2n-1\}$ which labels the edges by

$f^*(a_ia_{i+1})$ = Max of $\{f(a_i), f(a_{i+1})\}$

= Max of $\{2i-2, 2i\}$

= $2i$, for all i from 1 to n-1.

$f^*(a_ib_i)$ = Max of $\{f(a_i), f(b_i)\}$

= Max of $\{2i-2, 2i-1\}$

= $2i-1$, for all i from 1 to n.

gcin of (a_1) = gcd of $\{f^*(a_1b_1) , f^*(a_1a_2)$

= gcd of $\{1, 2\}$

= 1.

gcin of (ai+1) = gcd of $\{f^*(a_ia_{i+1}), f^*(a_{i+1}b_{i+1})\}$

= gcd of $\{2i, 2i+1\}$

= 1, for all i from 1 to n-1.

Hence $K_{1,n}$ is a maximal incidence edge prime graph.

Example 2.3 Maximal incidence edge prime labeling of $P_5 \odot K_1$ is shown in Figure 3

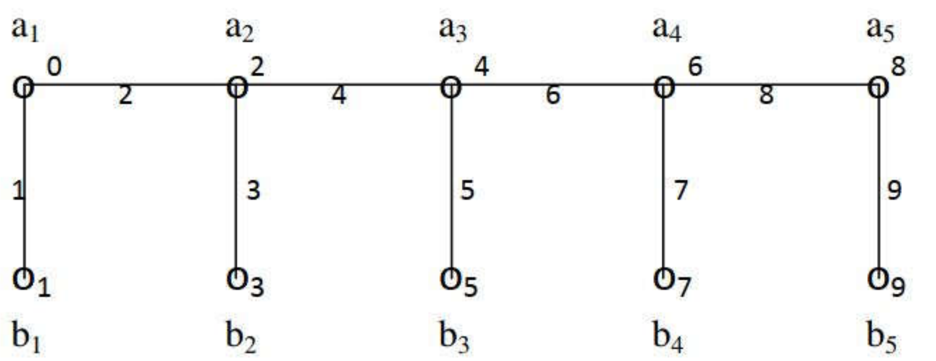

Figure 3

Theorem 2.4 Centipede graph $P_n \odot K_{1,2}$ is a maximal incidence edge prime graph.

Proof: Let $a_1, a_2, \ldots, a_n$, $b_1, b_2, \ldots, b_n$,$c_1, c_2, \ldots, c_n$,be the vertices of the graph.

Then $|V(P_n \odot K_{1,2})| = 3n$ and $|E(P_n \odot K_{1,2})| = 3n-1$.

I define a function $f : V \rightarrow \{0,1,\ldots,3n-1\}$ which labels the vertices by

$f(a_i) = 3i-2$ for all i from 1 to n.

$f(b_i) = 3i-3$ for all i from 1 to n.

$f(c_i) = 3i-1$ for all i from 1 to n.

I define a function f^*: E → {1,2,…,3n-1} which labels the edges by

$f^*(b_ib_{i+1})$ = Max of {f(b_i), f(b_{i+1})}
= Max of {3i-3, 3i}
= 3i , for all i from 1 to n-1.

$f^*(a_ib_i)$ = Max of {f(a_i), f(b_i)}
= Max of {3i-2, 3i-3}
= 3i-2, for all i from 1 to n.

$f^*(b_ic_i)$ = Max of {f(b_i), f(c_i)}
= Max of {3i-3, 3i-1}
= 3i-1, for all i from 1 to n.

gcin of (bi) = gcd of {$f^*(a_ib_i)$, $f^*(b_ic_i)$}
= gcd of {3i-2, 3i-1}
= 1, for all i from 1 to n.

Hence $P_n \odot K_{1,2}$ is a maximal incidence edge prime graph.

Example 2.4 Maximal incidence edge prime labeling of $P_4 \odot K_{1,2}$ is shown in Figure 4.

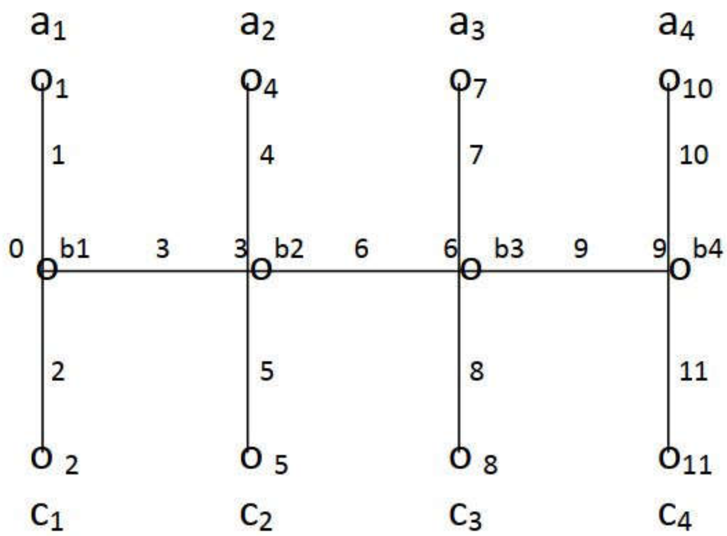

Figure 4.

Theorem 2.5 Coconut tree graph CT(m,n) is a maximal incidence edge prime graph.

Proof: Let $a_1,a_2,\ldots,a_m$, $b_1,b_2,\ldots,b_n$ be the vertices of the graph.

Then |V(CT(m,n))| = m+n and |E(CT(m,n))| = m+n-1.

I define a function f : V → {0,1,…,m+n-1} which labels the vertices by

f(a_i) = i-1 for all i from 1 to m.

f(b_i) = m+i-1 for all i from 1 to n.

I define a function f^*: E → {1,2,…,m+n-1} which labels the edges by

$f^*(a_ia_{i+1})$ = Max of {f(a_i), f(a_{i+1})}
= Max of {i-1, i}
= i , for all i from 1 to m-1.

$f^*(a_m b_i)$ = Max of $\{f(a_m), f(b_i)\}$
= Max of $\{m-1, m+i-1\}$
= $m+i-1$, for all i from 1 to n.

gcin of (a_m) = gcd of $\{f^*(a_{m-1}a_m), f^*(a_m b_1)\}$
= gcd of $\{m-1, m\}$
= 1.

gcin of (a_{i+1}) = gcd of $\{f^*(a_i a_{i+1}), f^*(a_{i+1}a_{i+2})\}$
= gcd of $\{i, i+1\}$
= 1, for all i from 1 to m-2.

Hence CT(m,n) is a maximal incidence edge prime graph.

Example 2.5 Maximal incidence edge prime labeling of CT(5,4) is shown in Figure 5.

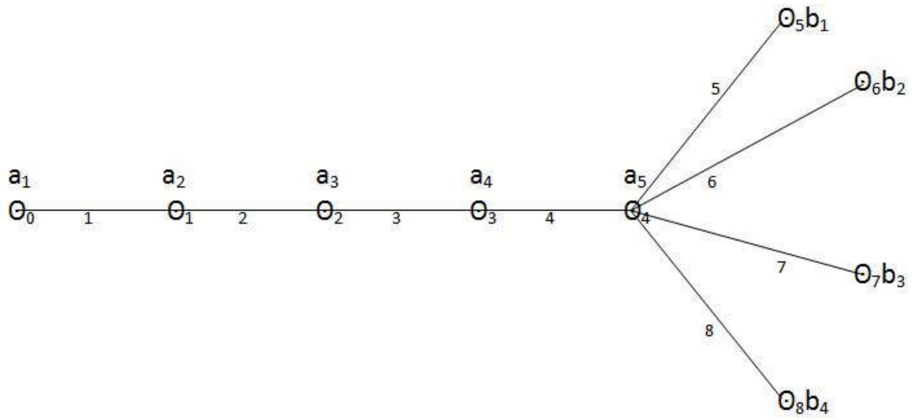

Figure 5.

Theorem 2.6 Bi star graph B(m,n) is a maximal incidence edge prime graph.

Proof: Let $x,y,a_1,a_2,\ldots,a_m, b_1,b_2,\ldots,b_n$ be the vertices of the graph. Then $|V(B(m,n))| = m+n+2$ and

$|E(B(m,n))| = m+n+1$.

I define a function $f : V \rightarrow \{0,1,\ldots,m+n+1\}$ which labels the vertices by

$f(x)$ = 0.
$f(y)$ = 1.
$f(a_i)$ = $i+1$ for all i from 1 to m.
$f(b_i)$ = $m+i+1$ for all i from 1 to n.

I define a function $f^*: E \rightarrow \{1,2,\ldots,m+n+1\}$ which labels the edges by

$f^*(xa_i)$ = max of $\{f(x), f(a_i)\}$
= max of $\{0, i+1\}$
= $i+1$, for all i from 1 to m.

$f^*(yb_i)$ = max of $\{f(y), f(b_i)\}$
= max of $\{1, m+i+1\}$
= $m+i+1$, for all i from 1 to n.

$f(xy)$ = 1.

gcin of (x) $= \gcd$ of $\{f^*(xa_1), f^*(xa_2)\}$
$= \gcd$ of $\{2, 3\}$
$= 1$.
gcin of (y) $= \gcd$ of $\{f^*(yb_1), f^*(yb_2)\}$
$= \gcd$ of $\{m+2, m+3\}$
$= 1$.

Hence B(m,n) is a maximal incidence edge prime graph.

Example 2.6 Maximal incidence edge prime labeling of B(3,4) is shown in Figure 6.

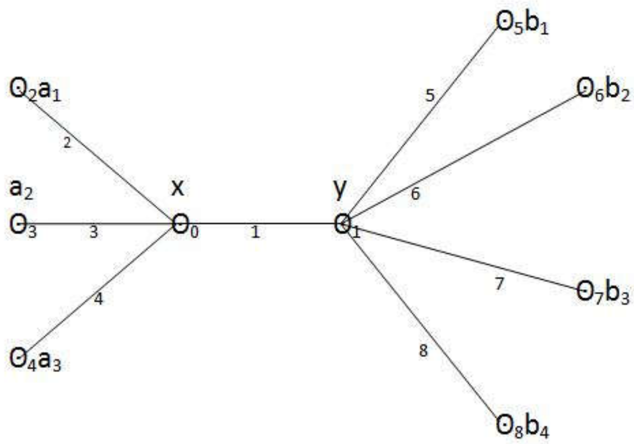

Figure 6

Theorem 2.7 Subdivision graph of star $K_{1,n}$($Sd(K_{1,n})$) is a maximal incidence edge prime graph.

Proof: Let $x, a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n$ be the vertices of the graph. Then $|V(Sd(K_{1,n}))| = 2n+1$ and $|E(Sd(K_{1,n}))| = 2n$.

I define a function $f : V \rightarrow \{0,1,\ldots,2n\}$ which labels the vertices by

$f(x) = 0$.
$f(a_i) = 2i-1$ for all i from 1 to n.
$f(b_i) = 2i$ for all i from 1 to n.

I define a function $f^*: E \rightarrow \{1,2,\ldots,2n\}$ which labels the edges by

$f^*(xa_i)$ = Max of $\{f(x), f(a_i)\}$
= Max of $\{0, 2i-1\}$
$= 2i-1$, for all i from 1 to n.
$f^*(a_ib_i)$ = Max of $\{f(a_i), f(b_i)\}$
= Max of $\{2i-1, 2i\}$
$= 2i$, for all i from 1 to n.
gcin of (a_i) $= \gcd$ of $\{f^*(xa_i), f^*(a_ib_i)\}$
$= \gcd$ of $\{2i-1, 2i\}$
$= 1$, for all i from 1 to n.
gcin of (x) $= \gcd$ of $\{f^*(xa_1), f^*(xa_2)\}$
$= \gcd$ of $\{1, 3\}$
$= 1$.

Hence $Sd(K_{1,n})$ is a maximal incidence edge prime graph.
Example 2.7 Maximal incidence edge prime labeling of $Sd(K_{1,4})$) is shown in Figure 7.

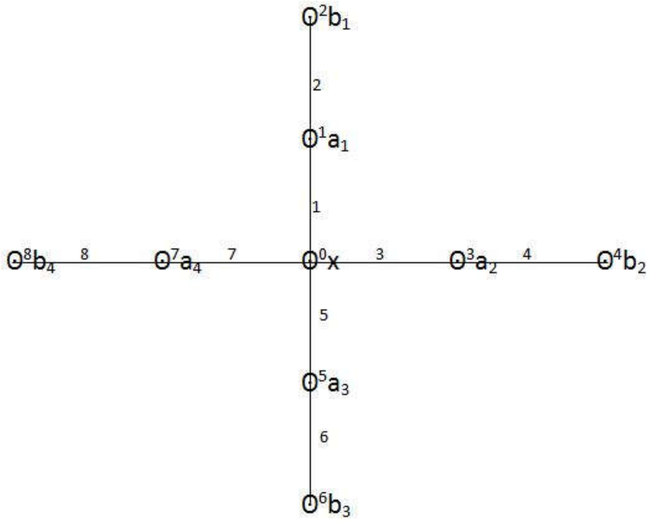

Figure 7.

Theorem 2.8 Twig graph $T_w(n)$ is a maximal incidence edge prime graph.
Proof: Let $a_1,a_2,\ldots,a_n$, $b_1,b_2,\ldots,b_{n-2}$,$c_1,c_2,\ldots,c_{n-2}$,be the vertices of the graph.
Then $|V(T_w(n))| = 3n-4$ and $|E(T_w(n))| = 3n-5$.
I define a function $f : V \to \{0,1,\ldots,3n-5\}$which labels the vertices by

$f(a_i)$ = i-1 for all i from 1 to n.

$f(b_i)$ = n+i-1, for all i from 1 to n-2.

$f(c_i)$ = 2n-3+I, for all i from 1 to n-2.

I define a functionf^*: $E \to \{1,2,\ldots,3n-5\}$ which labels the edges by

$f^*(a_ia_{i+1})$ = Max of $\{f(a_i), f(a_{i+1})\}$
= Max of $\{i-1, i\}$
= i , for all i from 1 to n-1.

$f^*(a_{i+1}b_i)$ = Max of $\{f(a_{i+1}), f(b_i)\}$
= Max of $\{i, n+i-1\}$
= n+i-1, for all i from 1 to n-2.

$f^*(a_{i+1}c_i)$ = Max of $\{f(a_{i+1}), f(c_i)\}$
= Max of $\{i, 2n-3+i\}$
= 2n-3+i, for all i from 1 to n-2.

gcin of (a_{i+1}) = gcd of $\{f^*(a_ia_{i+1}), f^*(a_{i+1}a_{i+2})\}$
= gcd of $\{i,i+1\}$
= 1, for all i from 1 to n-2.

Hence $T_w(n)$ is a maximal incidence edge prime graph.
Theorem 2.9 Sum of the edge values is equal to sum of the vertex values in a circuit less maximal incidence prime graph.

Proof : Let G be the graph with n vertices. Since G is a tree, it has only n-1 edges. Labels of the vertices are 0,1,…,n-1 and the labels of the edges are 1,2,…,n. Hence the theorem.
Example 2.8 Maximal incidence edge prime labeling of $T_w(5)$ is shown in Figure 8.

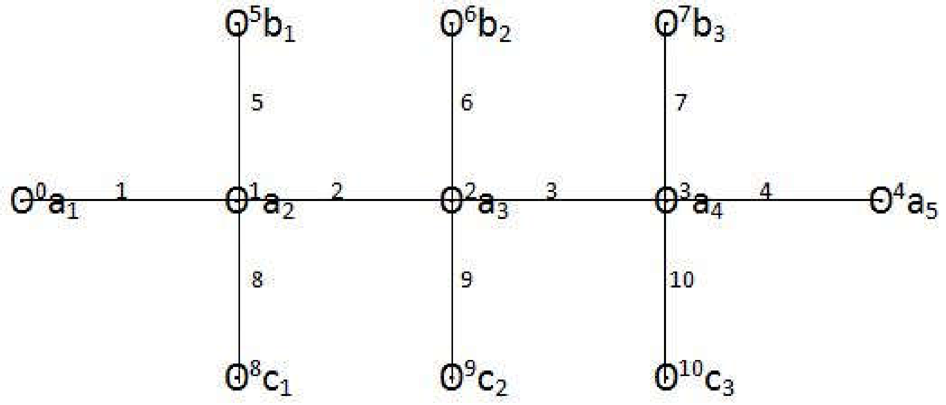

Figure 8.

3. CONCLUSION

In this paper I introduced a new labeling and proved that some connected circuit less graphs are maximal incidence edge prime graphs. Further I proved a general theorem on connected circuit less maximal incidence prime graphs. To explore some new maximal incidence edge prime graphs is an open problem

REFERENCES

[1] Apostol. Tom M, Introduction to Analytic Number Theory, Narosa, (1998).
[2] Harary F, Graph Theory, Addison-Wesley,Reading, Mass, (1972
[3] Joseph A Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics(2015), #DS6,Pages 1 – 389.
[4] Sunoj B S, Labeling Problems in Graph Theory, PhD Thesis, University of Kerala 2019.
[5] Sunoj B S, Mathew Varkey T K, "Square Sum Prime Labeling of Some Path Related Graphs", International Journal of Application or Innovation in Engineering & Management, Vol. 6, Issue 7, July 2017, pp 25-28.
[6] Sunoj B S, Mathew Varkey T K, "Square Difference Prime Labeling of Some Cycle Related Graphs", International Journal of Computational Engineering Research, Vol. 7, Issue 11, November 2017, pp 22-25.