New Similarity Analysis Method with Power Low Fluid with Consideration of Variable Physical Properties

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Abstract

A new similarity technique is applied to study of power low fluid model with consideration of variable physical properties. The governing partial Differential Equations for laminar forced convection flow are transformed to ordinary Differential Equations. Obtained velocity components $u_x(\emptyset)$ *and* $u_y(\emptyset)$ *and dimensionless temperature* $\theta(\emptyset)$ *describes the force convection momentum field with the coupled of effect of the variable physical properties on power low fluid is dominated by dimensionless physical property factors.*

Key Words: New Similarity Technique, Power – law model of variable physical properties, Non – Newtonian Fluid, Forced Convection, Reynolds Number, Prandtl Number, Nusselt Number.

Introduction

The new similarity analysis method is applied to develop a novel similarity analysis model of laminar forced convection with consideration of variable physical properties**.** Effect of the variable physical properties, such as density, thermal conductivity and viscosity on laminar liquid forced convection is reflected by the influence of the boundary temperature because these physical properties are temperature dependent. Increasing the boundary temperature differences $t_w - t_\infty$ for liquid laminar forced convection causes increase of the wall temperature gradient. Only the solution on case $t_w - t_\infty \to 0$ for liquid laminar forced convection with consideration of variable physical properties is identical to Blasius solution on the velocity field and Pohlhausen equation on heat transfer.In [1], a new similarity analysis method was proposed on extensive investigation for heat transfer of laminar free convection, and collected in [2].

In thispaper, a new similarity analysis method is reported for extensive investigation of boundary layer governing power low fluid with consideration of variable physical properties.

First, a system of dimensionless similarity variables, such as Reynolds number, dimensionless coordinate variable and dimensionless velocity components, is derived and determined through the analysis with the typical basis conservation equations In derivation of the dimensionless similarity variables, it is never necessary to induce the stream function ψ,intermediate variable $f(\emptyset)$, and its derivatives.

In this way, we attempt to determine the similarity solution for the problem of Power low Fluid with consideration of variable physical properties.

Governing Equation

We suppose the laminar flow of Power low Fluid with consideration of variable physical properties. Many authors, as Balsius ordinary equation transformed from the momentum partial differential equations [3]. Pohlhausen [4] used Balsius transformation system to further calculate heat transfer with constant property assumption for laminar forced convection on a horizontal flat plate. Schlichting [5] investigated the effect of the variable thermo physical properties on compressible forced boundary layer. For incompressible forced boundary layer, there have been some studies with consideration of variable liquid viscosity, the studies of Ling and Dybbs [6, 7]. Similarity solutions of two-dimensional conservation equation for non-Newtonian laminar forced convection boundary layer on a horizontal flat plate was investigated using one-parameter linear group of transformation by Surati and Timol [8]. Hema C. Surati and M.G.Timol have derived Deductive Group Method for Heat Transfer of Gas Laminar Forced Convection Flow with Variable Physical Property [9].

Governing Partial Differential Equations

A flat plate is horizontally located in parallel fluid flow with its main steam velocity $u_{x,\infty}$. The plate surface temperature is t_w and the liquid bulk temperature is t_∞. Then a velocity boundary layer is produced near the plate. If t_w is not equal to t_∞ , a temperature boundary layer will occur near the plate also. We assume that the velocity boundary layer is laminar. Then the governing partial differential equations for power low fluid expressed as below with consideration of these variable physical properties:

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0 \tag{1}$$

$$\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=\mu\rho^{n-1}\left\{\frac{\partial}{\partial y}\left|\frac{\partial u}{\partial y}\right|^{n}\right\} \tag{2}$$

$$\rho\left[u\frac{\partial(c_p t)}{\partial x}+\frac{\partial(c_p t)}{\partial y}=\frac{\partial}{\partial y}\left(\lambda\frac{\partial t}{\partial y}\right)\right] \tag{3}$$

With the boundary conditions

$$y=0: u=0, v=0, t=t_w \tag{4}$$

$$y\to\infty: u=U(constant), t=t_\infty \tag{5}$$

Here, temperature dependent physical properties density ρ, absolute viscosity μ, thermal conductivity λ, and specific heat c_p are taken into account.

Similarity Transformation Variables

The similarity transformation variables used for laminar forced convections without consideration of variable physical properties can be taken as those with consideration of variable physical properties in this problem. The average value on physical properties should be changed to the related local value and therefore, the local Reynolds number $Re = \frac{Ux}{\vartheta}$ with the average kinematic viscosity ϑ is changed to the local Reynolds number $Re = \frac{Ux}{\vartheta_\infty}$ with the local kinetic viscosity ϑ_∞. In this case, for the new similarity analysis method, a set of the related dimensional analysis variables can be shown as follows for consideration of variable physical properties. all these dimensionless quantities are derived in my paper [10].

$$\emptyset = \frac{y}{x}\left(\frac{1}{2}Re_\infty\right)^{\frac{1}{2}} \tag{6}$$

$$Re = \frac{Ux}{\vartheta_\infty} \tag{7}$$

$$u = U\, u_x(\emptyset) \tag{8}$$

$$v = U\left(\frac{1}{2}Re_\infty\right)^{-\frac{1}{2}} u_y(\emptyset) \tag{9}$$

$$\theta(\emptyset) = \frac{t - t_\infty}{t_w - t_\infty} \tag{10}$$

When $\emptyset$ is dimensionless coordinate variable, Re is local Reynolds number with consideration of variable physical properties, $u_x(\emptyset)$ and $u_y(\emptyset)$ are dimensionless velocity components in x and y coordinates, respectively and $\theta(\emptyset)$ denotes dimensionless temperature.

Similarity Transformation of the Governing partial Differential Equations

Transformation of (1)

Equation (1) is changed to the following form

$$\rho\frac{\partial u}{\partial x} + \rho\frac{\partial v}{\partial y} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} = 0 \tag{11}$$

With we have

$$\frac{\partial u}{\partial x} = U\frac{du_x(\emptyset)}{d\emptyset}\frac{\partial \emptyset}{\partial x}$$

Where with (6) we have

$$\frac{d\emptyset}{dx} = -\frac{1}{2}\emptyset\, x^{-1} \tag{12}$$

Then,

$$\frac{\partial u}{\partial x} = -\frac{1}{2}\, \emptyset\, \mathrm{x}^{-1} U \frac{du_x(\emptyset)}{d\emptyset} \quad (13)$$

With (9) we have

$$\frac{\partial v}{\partial y} = U\left(\frac{1}{2}\, Re\right)^{-\frac{1}{2}} \frac{du_y(\emptyset)}{d\emptyset}\, \frac{\partial \emptyset}{\partial y}$$

Where

$$\frac{\partial \emptyset}{\partial y} = \mathrm{x}^{-1}\, \left(\frac{1}{2}\, Re_\infty\right)^{\frac{1}{2}} \quad (14)$$

Then

$$\frac{\partial v}{\partial y} = U \frac{du_y(\emptyset)}{d\emptyset}\, \mathrm{x}^{-1} \quad (15)$$

In addition,

$$\frac{\partial \rho}{\partial \mathrm{x}} = \frac{\mathrm{d}\rho}{\mathrm{d}\emptyset}\, \frac{\partial \emptyset}{\partial x} = -\frac{1}{2}\, \frac{\mathrm{d}\rho}{\mathrm{d}\emptyset}\, \emptyset \mathrm{x}^{-1} \quad (16)$$

$$\frac{\partial \rho}{\partial \mathrm{y}} = \frac{\mathrm{d}\rho}{\mathrm{d}\emptyset}\, \frac{\partial \emptyset}{\partial y} = \frac{\mathrm{d}\rho}{\mathrm{d}\emptyset} \mathrm{x}^{-1}\left(\frac{1}{2}\, Re_\infty\right)^{\frac{1}{2}} \quad (17)$$

Use (13), (15), (16) and (17) are used in (11), we have

$$\rho\left[-\frac{1}{2}\, \emptyset\, \mathrm{x}^{-1} U \frac{du_x(\emptyset)}{d\emptyset} + U \frac{du_y(\emptyset)}{d\emptyset}\, \mathrm{x}^{-1}\right] - \frac{1}{2}\, \frac{\mathrm{d}\rho}{\mathrm{d}\emptyset}\, \emptyset \mathrm{x}^{-1} \mathrm{U}\, u_x(\emptyset) +$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}\emptyset} \mathrm{x}^{-1}\left(\frac{1}{2}\, Re_\infty\right)^{\frac{1}{2}} U\, \left(\frac{1}{2}\, Re_\infty\right)^{-\frac{1}{2}}\, u_y(\emptyset) = 0$$

The above equation is divided by $\frac{1}{2}\, \mathrm{x}^{-1}\mathrm{U}$ and we have

$$\rho\left[-\, \emptyset\, \frac{du_x(\emptyset)}{d\emptyset} + 2 \frac{du_y(\emptyset)}{d\emptyset}\right] - \, \frac{\mathrm{d}\rho}{\mathrm{d}\emptyset}\, \emptyset\, u_x(\emptyset) + 2 \frac{\mathrm{d}\rho}{\mathrm{d}\emptyset}\, u_y(\emptyset) = 0$$

The above equation is simplified to

$$-\emptyset\, \frac{du_x(\emptyset)}{d\emptyset} + 2 \frac{du_y(\emptyset)}{d\emptyset} + \frac{1}{\rho}\, \frac{\mathrm{d}\rho}{\mathrm{d}\emptyset}\left[-\emptyset\, u_x(\emptyset) + 2u_y(\emptyset)\right] = 0 \quad (18)$$

Transformation of (2)

Take derivative of (8) with respect to y,

$$\frac{\partial u}{\partial y} = Ux^{-1}\frac{du_x(\emptyset)}{d\emptyset}\left(\frac{1}{2}Re_\infty\right)^{\frac{1}{2}} \tag{19}$$

$$\left|\frac{\partial u}{\partial y}\right|^n = U^n x^{-n}\left(\frac{du_x(\emptyset)}{d\emptyset}\right)^n \left(\frac{1}{2}Re_\infty\right)^{\frac{n}{2}}$$

$$\frac{\partial}{\partial y}\left|\frac{\partial u}{\partial y}\right|^n = nU^n x^{-n-1}\left(\frac{du_x(\emptyset)}{d\emptyset}\right)^{n-1}\left(\frac{1}{2}Re_\infty\right)^{\frac{n+1}{2}}\frac{d^2u_x(\emptyset)}{d\emptyset^2} \tag{20}$$

(8), (9), (13), (19)and (20) are use in (2), we have

$$\rho\left\{U^2x^{-1}u_x(\emptyset)\left(-\frac{\emptyset}{2}\right)\frac{du_x(\emptyset)}{d\emptyset} + U^2x^{-1}u_y(\emptyset)\frac{du_x(\emptyset)}{d\emptyset}\right\} =$$
$$\left\{\mu n x^{-n-1}U^n\left(\frac{1}{2}Re_\infty\right)^{\frac{n+1}{2}}\left(\frac{du_x(\emptyset)}{d\emptyset}\right)^{n-1}\frac{d^2u_x(\emptyset)}{d\emptyset^2}\right\} + \left\{U^n\, x^{-1-n}\left(\frac{1}{2}Re_\infty\right)^{\frac{1}{2}+\frac{n}{2}}\left(\frac{du_x(\emptyset)}{d\emptyset}\right)^n\frac{d\mu}{d\emptyset}\right\}$$

The above equation divide both the sides by U^2x^{-1} and simplify it,

$$\rho\left\{u_x(\emptyset)\left(-\frac{\emptyset}{2}\right)\frac{du_x(\emptyset)}{d\emptyset} + u_y(\emptyset)\frac{du_x(\emptyset)}{d\emptyset}\right\}$$
$$= \frac{x^{\frac{-n+1}{2}}U^{\frac{3n-3}{2}}\left(\frac{1}{2}\right)^{\frac{n+1}{2}}}{\vartheta_\infty^{\frac{n+1}{2}}}\left\{\mu n\left(\frac{du_x(\emptyset)}{d\emptyset}\right)^{n-1}\frac{d^2u_x(\emptyset)}{d\emptyset^2} + \left(\frac{du_x(\emptyset)}{d\emptyset}\right)^n\frac{d\mu}{d\emptyset}\right\}$$

Consider $k = \dfrac{x^{\frac{-n+1}{2}}U^{\frac{3n-3}{2}}\left(\frac{1}{2}\right)^{\frac{n+1}{2}}}{\vartheta_\infty^{\frac{n+1}{2}}}$

$$\rho\left\{u_x(\emptyset)\left(-\frac{\emptyset}{2}\right)\frac{du_x(\emptyset)}{d\emptyset} + u_y(\emptyset)\frac{du_x(\emptyset)}{d\emptyset}\right\} = k\left\{\mu n\left(\frac{du_x(\emptyset)}{d\emptyset}\right)^{n-1}\frac{d^2u_x(\emptyset)}{d\emptyset^2} + \left(\frac{du_x(\emptyset)}{d\emptyset}\right)^n\frac{d\mu}{d\emptyset}\right\} \tag{21}$$

Transformation of (3)

Equation (3) is changed to the following form

$$\rho\left(uc_p\frac{\partial t}{\partial x} + u\,t\,\frac{\partial c_p}{\partial x} + v\,c_p\frac{\partial t}{\partial y} + v\,t\,\frac{\partial c_p}{\partial y}\right) = \lambda\,\frac{\partial^2 t}{\partial y^2} + \frac{\partial\lambda}{\partial x}\frac{\partial t}{\partial y} \tag{22}$$

Use (6), (10)and (12), we get

$$\frac{\partial t}{\partial x} = (t_w - t_\infty)\,\frac{d\theta(\emptyset)}{d\emptyset}\frac{\partial\emptyset}{\partial x}$$

$$\therefore \frac{\partial t}{\partial x} = -\frac{1}{2} \emptyset \text{x}^{-1} (t_w - t_\infty) \frac{d\theta(\emptyset)}{d\emptyset} \quad (23)$$

Use (6),(10) and (14), we get

$$\frac{\partial t}{\partial y} = (t_w - t_\infty) \frac{d\theta(\emptyset)}{d\emptyset} \frac{\partial \emptyset}{\partial y}$$

$$\frac{\partial t}{\partial y} = \text{x}^{-1} \left(\frac{1}{2} Re_\infty\right)^{\frac{1}{2}} (t_w - t_\infty) \frac{d\theta(\emptyset)}{d\emptyset} \quad (24)$$

$$\frac{\partial^2 t}{\partial y^2} = \text{x}^{-2} \left(\frac{1}{2} Re_\infty\right) (t_w - t_\infty) \frac{d^2\theta(\emptyset)}{d\emptyset^2} \quad (25)$$

$$\frac{\partial c_p}{\partial x} = -\frac{1}{2} \frac{dc_p}{d\emptyset} \emptyset \text{x}^{-1} \quad (26)$$

$$\frac{\partial \lambda}{\partial y} = \frac{d\lambda}{d\emptyset} \text{x}^{-1} \left(\frac{1}{2} Re_\infty\right)^{\frac{1}{2}} \quad (27)$$

$$\frac{\partial c_p}{\partial y} = \frac{dc_p}{d\emptyset} \text{x}^{-1} \left(\frac{1}{2} Re_\infty\right)^{\frac{1}{2}} \quad (28)$$

(8), (9), (25) to (28) are use in (22) and changed to

$$\rho[U\, u_x(\emptyset)\, c_p \left(-\frac{1}{2} \emptyset \text{x}^{-1} (t_w - t_\infty) \frac{d\theta(\emptyset)}{d\emptyset}\right) + U u_x(\emptyset)\, t \left(-\frac{1}{2} \emptyset \text{x}^{-1} \frac{dc_p}{d\emptyset}\right)$$

$$+U\, u_y(\emptyset)\, c_p \text{x}^{-1} (t_w - t_\infty) \frac{d\theta(\emptyset)}{d\emptyset} + U u_y(\emptyset)\, t \frac{dc_p}{d\emptyset} \text{x}^{-1}]$$

$$= \lambda\, \text{x}^{-2} \left(\frac{1}{2} Re_\infty\right) (t_w - t_\infty) \frac{d^2\theta(\emptyset)}{d\emptyset^2} + \frac{d\lambda}{d\emptyset} \text{x}^{-1} \left(\frac{1}{2} Re_\infty\right) \text{x}^{-1} (t_w - t_\infty) \frac{d\theta(\emptyset)}{d\emptyset}$$

Use the definition of Re_∞ in above equation, we have

$$\rho[U\, u_x(\emptyset)\, c_p \left(-\frac{1}{2} \emptyset \text{x}^{-1} (t_w - t_\infty) \frac{d\theta(\emptyset)}{d\emptyset}\right) + U u_x(\emptyset)\, t \left(-\frac{1}{2} \emptyset \text{x}^{-1} \frac{dc_p}{d\emptyset}\right)$$

$$+U\, u_y(\emptyset)\, c_p \text{x}^{-1} (t_w - t_\infty) \frac{d\theta(\emptyset)}{d\emptyset} + U u_y(\emptyset)\, t \frac{dc_p}{d\emptyset} \text{x}^{-1}]$$

$$= \lambda\, \text{x}^{-2} \left(\frac{1}{2} \frac{Ux}{\vartheta_\infty}\right) (t_w - t_\infty) \frac{d^2\theta(\emptyset)}{d\emptyset^2} + \frac{d\lambda}{d\emptyset} \text{x}^{-1} \left(\frac{1}{2} \frac{Ux}{\vartheta_\infty}\right) \text{x}^{-1} (t_w - t_\infty) \frac{d\theta(\emptyset)}{d\emptyset} \quad (29)$$

The above equation is divided by $\frac{1}{2}(t_w - t_\infty) \frac{c_p\, U\lambda}{x\vartheta_\infty}$ and changed to

Where, $\frac{t}{t_w - t_\infty} = \frac{(t_w - t_\infty)\,\theta(\emptyset) + t_\infty}{t_w - t_\infty} = \theta(\emptyset) + \frac{t_\infty}{t_w - t_\infty}$ (30)

Use (30) in (29) and simplify it,

$$\Pr \frac{v_\infty}{\vartheta}\left\{\left(-\emptyset u_x(\emptyset) + 2u_y(\emptyset)\right)\frac{d\theta(\emptyset)}{d\emptyset}\left(-\emptyset u_x(\emptyset) + 2u_y(\emptyset)\right)\left(\theta(\emptyset) + \frac{t_\infty}{t_w - t_\infty}\right)\frac{1}{c_p}\frac{dc_p}{d\emptyset}\right\}$$

$$= \frac{d^2\theta(\emptyset)}{d\emptyset^2} + \frac{1}{\lambda}\frac{d\lambda}{d\emptyset}\frac{d\theta(\emptyset)}{d\emptyset}$$

For summary, governing partial differential equations (1), (2) and (3) have been transformed to the following governing ordinary differential equations:

$$-\emptyset\,\frac{du_x(\emptyset)}{d\emptyset} + 2\frac{du_y(\emptyset)}{d\emptyset} + \frac{1}{\rho}\frac{\partial\rho}{\partial\emptyset}\left[-\emptyset\, u_x(\emptyset) + 2u_y(\emptyset)\right] = 0$$

$$\rho\left\{u_x\left(-\frac{\emptyset}{2}\right)\frac{du_x(\emptyset)}{d\emptyset} + u_y\frac{du_x(\emptyset)}{d\emptyset}\right\} = \; k\left\{\mu n\left(\frac{du_x(\emptyset)}{d\emptyset}\right)^{n-1}\frac{d^2u_x(\emptyset)}{d\emptyset^2} + \left(\frac{du_x(\emptyset)}{d\emptyset}\right)^n\frac{d\mu}{d\emptyset}\right\}$$

$$\Pr \frac{v_\infty}{\vartheta}\left\{\left(-\emptyset u_x(\emptyset) + 2u_y(\emptyset)\right)\frac{d\theta(\emptyset)}{d\emptyset}\left(-\emptyset u_x(\emptyset) + 2u_y(\emptyset)\right)\left(\theta(\emptyset) + \frac{t_\infty}{t_w - t_\infty}\right)\frac{1}{c_p}\frac{dc_p}{d\emptyset}\right\} = \frac{d^2\theta(\emptyset)}{d\emptyset^2} + \frac{1}{\lambda}\frac{d\lambda}{d\emptyset}\frac{d\theta(\emptyset)}{d\emptyset}$$

Boundary condition equations (4) and (5) are transformed to

$$\emptyset = 0{:} \Rightarrow u_x(\emptyset) = 0,\, u_y(\emptyset) = 0,\, \theta(\emptyset) = 1$$

$$\emptyset \to \infty{:} \Rightarrow u_x(\emptyset) = 1,\, \theta(\emptyset) = 0$$

Equations (1), (2) and (3) are complete ordinary differential equations of laminar forced convection power low fluid. These transformed governing ordinary differential equations are completely dimensionless because of they involve dimensionless velocity components $u_x(\emptyset)$ and $u_y(\emptyset)$, dimensionless temperature $\theta(\emptyset)$, all their derivatives, all physical properties exits in form of the dimensionless physical property factors, such as $\frac{1}{\rho}\frac{\partial\rho}{\partial\emptyset}$, $\frac{1}{\mu}\frac{d\mu}{d\emptyset}$, $\frac{1}{\lambda}\frac{d\lambda}{d\emptyset}$, $\frac{1}{c_p}\frac{dc_p}{d\emptyset}$, $\frac{v_\infty}{\vartheta}$ and $\frac{t_\infty}{t_w - t_\infty}$. The coupled of effect of the variable physical properties on power low fluid is dominated by these dimensionless physical property factors.

Conclusion

A set of dimensionless similarity mathematical models of Power low Fluid with consideration of variable physical properties is developed by the new similarity analysis method. With this new similarity analysis method, the two dimensional velocity components are directly transformed to the related dimensionless velocity components, which lead to a convenience for dealing with the variable physical properties.

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