A STOCHASTIC MODEL TO ESTIMATE THE EXPECTED TIME TO RECRUITMENT IN SINGLE GRADE MANPOWER SYSTEM USING SHIFTED EXPONENTIAL DISTRIBUTION

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Abstract

In this paper shock model approach to estimate the threshold level is used derive by the expected time to recruitment in an organization using for the threshold level shifted exponential distribution, the recruitment takes place whenever the cumulative loss of man hour exceeds the threshold. The findings of the study suggest that the expected time to reach the recruitment status of the organization and its variance is found, depend upon the different input factors are the parameters of the distribution involved. The examples obtained are illustrated with suitable numerical examples.

Keywords: Shifted Exponential Distribution, Recruitment, Organization, Expected Time, Variance and Threshold.

1. INTRODUCTION

Human resource management is an important application oriented division of management science. In many industries or organization to leaver resaved which is also have as attrition is unavailable scheme are many reason for the personal to leave the organization revision or policies regarding work schedule, pay scale etc., are often to avoid the complication due to attrition the organization going recruitment will have impact on the financial condition work schedule and also the goodness or fit organization so as a in after or policy its recruitment process is unprompted only depression of manpower due to successive policy decision critical so called threshold level.

In this paper the expected time to recruitment which is also the expected time to cross the threshold level or depletions derived under the assumption that the threshold level is random variable which follows shifted exponential distribution numerical illustration also provided.

Mathematical models are obtained for the expected time to reach the threshold level under various assumptions by several authors. One can see for more details in Esary *et.al.,* [2], Sathiyamoorthi [6], Pandiyan *et.al.,* [5], Elangovan *et.al.,* [1] and Kannadasan *et.al.,*[3] about the expected level of the loss of manpower.

ASSUMPTIONS OF THE MODELS

- Exit of person from an organization takes place whenever the policy decisions regarding targets, incentives and promotions are made.
- The exit of every person from the organization results in a random amount of depletion of manpower (in man hours).
- The process of depletion is linear and cumulative.
- The inter arrival times between successive occasions of wastage are i.i.d. random variables.
- If the total depletion exceeds a threshold level Y which is itself a random variable, the breakdown of the organization occurs. In other words recruitment becomes inevitable.
- The process, which generates the exits, the sequence of depletions and the threshold are mutually independent.

NOTATIONS

X_i :a continuous random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the i[th] occasion of policy announcement, $i = 1,2,3, \dots k$ and $X_i^{'}s$ are i.i.d and $X_i = X$ for all i.

Y : a continuous random variable denoting the threshold level having the Shifted Exponential Distribution.

$g(.)$: The probability density functions (p.d.f) of X_i

$g_k(.)$: The k- fold convolution of $g(.)$ i.e., p.d.f. of $\sum_{i=1}^{k} X_i$

$g*(.)$: Laplace transform of $g(.)$; $g_k^*(.)$: Laplace transform of $\mathrm{g_k}(.)$

$h(.)$: The probability density functions of random threshold level which has the Shifted Exponential Distribution and $H(.)$is the corresponding probability generating functions.

U : a continuous random variable denoting the inter-arrival times between decision epochs.

$f(.)$: p.d.f. of random variable U with corresponding Probability generating function.

$V_k(t)$: $F_k(t) - F_{k+1}(t)$

$F_k(t)$: Probability that there are exactly 'k' policies decisions in (0, t]
$S(.)$: The survivor function i.e. $P[T > t]$;$1 - S(t) = L(t)$

2. MODEL DESCRIPTION AND SOLUTION

The Probability Density Function (PDF) of the Shifted Exponential Distribution is:

$$f(x) = \frac{1}{\beta} e^{-\left(\frac{x-d}{\beta}\right)} \qquad x \geq d$$

The Cumulative Distribution Function (CDF) of the Distribution is

$$F(x) = \left[1 - e^{-\left(\frac{x-d}{\beta}\right)}\right] \qquad x \geq d$$

The corresponding Survival Function is (SF)

$$\overline{H} = 1 - F(x)$$

$$= e^{-\left(\frac{x-d}{\beta}\right)}$$

There may be no practical way to inspect an individual item to determine its threshold Y. in this case; the threshold must be a random variable. In general that the threshold Y follows Shifted Exponential Distribution with parameter β. It can be shown that,

$$P(X_i < y) = \int_0^\infty g_k(x)\overline{H}(x)\,dx$$

$$= \int_0^\infty g_k(x) e^{-\left(\frac{x-d}{\beta}\right)}$$

$$= \int_0^\infty g_k^*(x) e^{-\left(\frac{x-d}{\beta}\right)} dx$$

On simplifications we get,

$$= \left[g^{*}\left(\frac{1-d}{\beta} \right) \right]^{k}$$

The survival function which gives the probability that the cumulative threshold will fail only after time t.

$S(t) = P(T > t) =$ Probability that the total damage survives beyond t

$$= \sum_{k=0}^{\infty} P\,\{\text{there are exactly k decisions in } (0,t]$$

$$* \; P\,(\text{the total cumulative threshold } (0,t]\}$$

The survival function $S(t)$ which is the probability that an individual survives for a time t

It is also known from renewal process that

$$P(T > t) = \sum_{k=0}^{\infty} F_k(t) P(X_i < y)$$

$$= \sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t) \right] \left[g^{*}\left(\frac{1-d}{\beta} \right) \right]^{k}$$

$$= \left[1 - g^{*}\left(\frac{1-d}{\beta} \right) \right] \sum_{k=1}^{\infty} F_k(t) \left[g^{*}\left(\frac{1-d}{\beta} \right) \right]^{k-1}$$

Now, the life time is given by

$P(T < t) = L(t) =$ the distribution function of life time (t)

Using convolution theorem for Laplace transforms, $F_0(t) = 1$ and on simplification, it can shown that,

$$L(T) = 1 - S(t)$$

Taking Laplace transformation $L(T)$, we get,

$$=1-\left[1-g^{*}\left(\frac{1-d}{\beta}\right)\right]\sum_{k=1}^{\infty}F_{k}\left(t\right)\left[g^{*}\left(\frac{1-d}{\beta}\right)\right]^{k-1}$$

By taking Laplace-Stieltjes transform, it can be shown that

$$l^{*}\left(s\right)=\frac{\left[1-g^{*}\left(\frac{1-d}{\beta}\right)\right]f^{*}\left(s\right)}{\left[1-g^{*}\left(\frac{1-d}{\beta}\right)f^{*}\left(s\right)\right]}$$

Let the random variable U denoting inter arrival time which follows exponential with parameter. Now $f^{*}\left(s\right)=\left(\frac{c}{c+s}\right)$, substituting in the above equation we get,

$$=\frac{\left[1-g^{*}\left(\frac{1-d}{\beta}\right)\right]\frac{c}{c+s}}{\left[1-g^{*}\left(\frac{1-d}{\beta}\right)\frac{c}{c+s}\right]}$$

$$=\frac{\left[1-g^{*}\left(\frac{1-d}{\beta}\right)\right]c}{\left[c+s-g^{*}\left(\frac{1-d}{\beta}\right)c\right]}$$

$$E\left(T\right)=-\frac{d}{ds}l^{*}\left(s\right)\text{ given }s=0$$

$$=\frac{1}{\left[1-g^{*}\left(\frac{1-d}{\beta}\right)\right]c}$$

$$E\left(T^{2}\right)=\frac{d^{2}}{ds^{2}}l^{*}\left(s\right)\text{ given }s=0$$

$$= \frac{1}{\left[1 - g^*\left(\frac{1-d}{\beta}\right)\right]^2 c^2}$$

$$g^*\left(\frac{1-d}{\beta}\right) \sim \left(\frac{\mu}{\mu + \left(\frac{1-d}{\beta}\right)}\right)$$

$$E(T) = \frac{[1 + \mu\beta - d]}{c[1-d]}$$

$$E(T^2) = \frac{[1 + \mu\beta - d]^2}{c^2[1-d]^2}$$

From which $V(T)$ can be obtained

$$V(T) = E(T^2) - [E(T)]^2$$

$$= \frac{[1 + \mu\beta - d]^2}{c^2[1-d]^2} - \left[\frac{[1 + \mu\beta - d]}{c[1-d]}\right]^2$$

$$V(T) = \frac{[1 + \mu\beta - d]^2\left[1 - \frac{1}{\beta}\right]}{c^2[1-d]^2}$$

3. Numerical Illustrations

The mean and variance of time to recruitment is numerically illustrated by varying one parameter and keeping other parameters fixed. The effect of the parameters d, β, μ and c on the performance measures is shown in the following table.

c	μ =2	μ=4	μ=6	μ=8
1	0.4	1.2	2	2.8
2	0.1	0.3	0.5	0.7
3	0.044	0.133	0.222	0.311
4	0.025	0.075	0.125	0.175
5	0.016	0.048	0.08	0.112
6	0.011	0.033	0.056	0.078
7	8.163*10^-3	0.024	0.041	0.057
8	6.25*10^-3	0.019	0.031	0.044
9	4.938*10^-3	0.015	0.025	0.035
10	4*10^-3	0.012	0.02	0.028

Table 1: Effect of μ on the performance measures E (T)

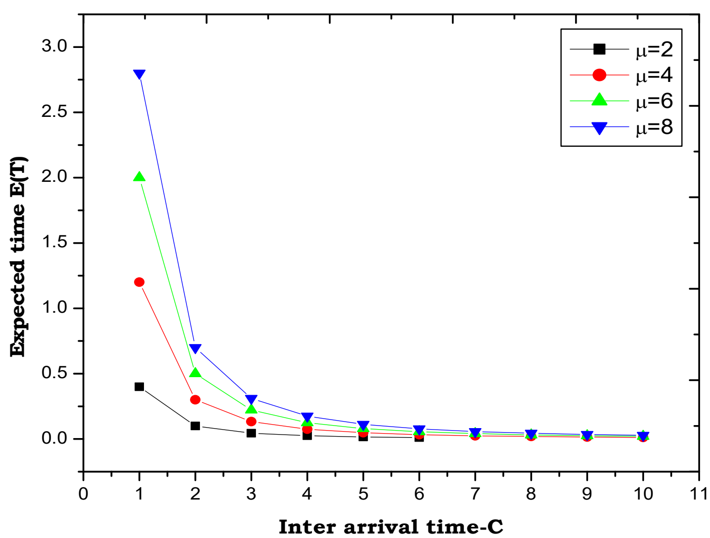

Figure 1: The changes in E (T) due to changes in μ

c	μ=2	μ=4	μ=6	μ=8
1	0.24	2.16	6	11.76
2	0.06	0.54	1.5	2.94
3	0.027	0.24	0.667	1.307
4	0.015	0.135	0.375	0.735
5	0.01	0.086	0.24	0.47
6	0.006	0.06	0.167	0.327
7	0.004	0.044	0.122	0.24
8	0.0037	0.034	0.094	0.184
9	0.0029	0.027	0.074	0.145
10	0.0024	0.022	0.06	0.118

Table 2: Effect of μ on the performance measures E (T)

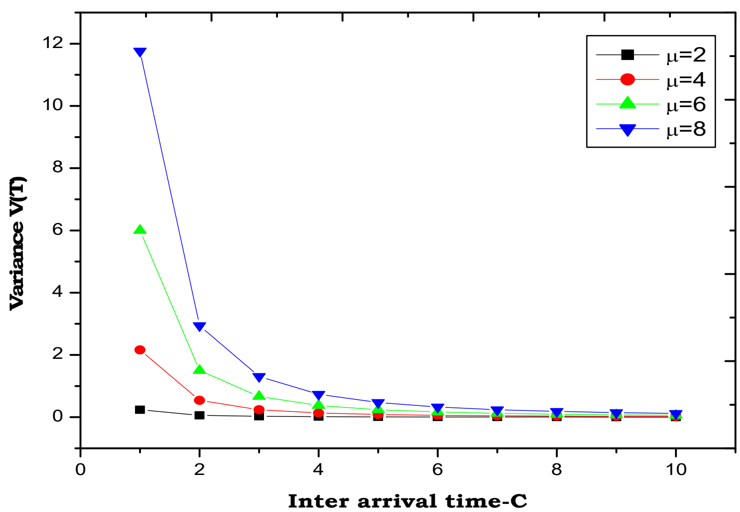

Figure 2: The changes in V (T) due to changes in μ

c	d=2	d=4	d=6	d=8
1	1.1	0.1	0.1	0.186
2	0.275	0.025	0.025	0.046
3	0.122	0.011	0.011	0.021
4	0.069	0.006	0.006	0.012
5	0.044	0.004	0.004	0.007
6	0.031	0.003	0.003	0.005
7	0.022	0.002	0.002	0.004
8	0.017	0.0015	0.002	0.003
9	0.014	0.0012	0.0012	0.002
10	0.011	0.001	0.001	0.0018

Table 3: Effect of d on the performance measures E (T)

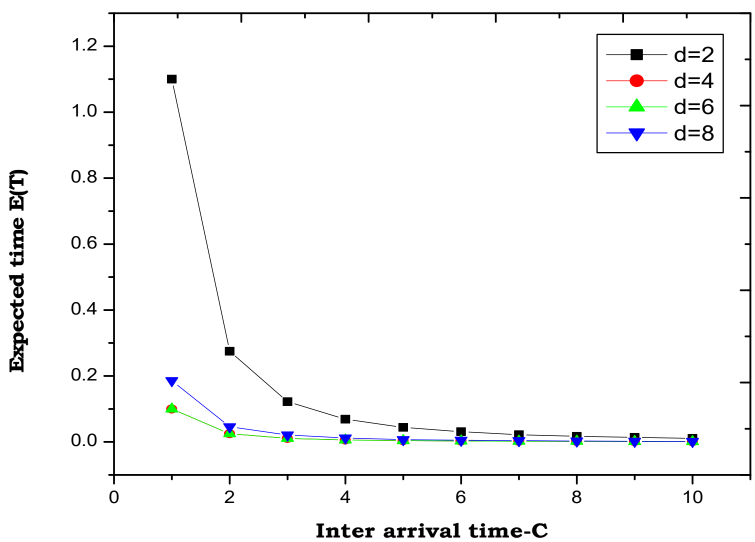

Figure 3: The changes in E (T) due to changes in d

c	d=2	d=4	d=6	d=8
1	1.815	0.015	0.015	0.052
2	0.454	0.004	0.004	0.013
3	0.202	0.002	0.002	0.006
4	0.113	0.0009	0.001	0.003
5	0.073	0.0006	0.0006	0.002
6	0.05	0.0004	0.0004	0.0014
7	0.037	0.0003	0.0003	0.001
8	0.028	0.0023	0.0023	0.0008
9	0.022	0.0018	0.00018	0.0006
10	0.018	0.0002	0.00015	0.0004

Table 4: Effect of d on the performance measures V (T)

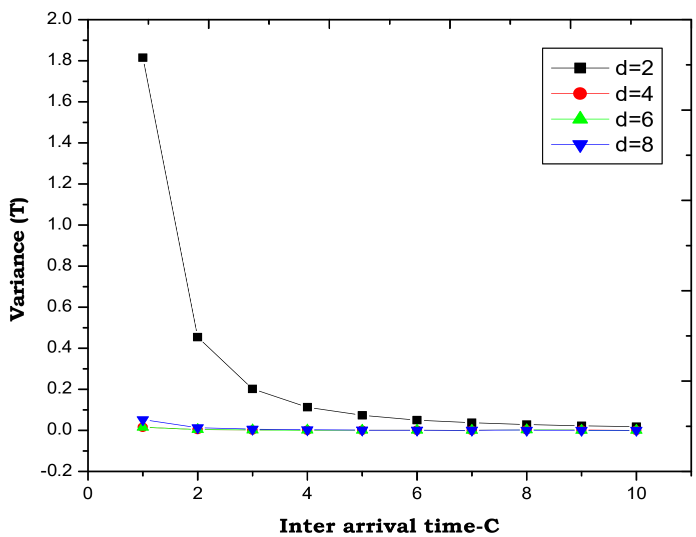

Figure 4: The changes in V (T) due to changes in d

c	β=2	β=4	β=6	β=8
1	0.1	0.35	0.433	0.475
2	0.025	0.088	0.108	0.119
3	0.011	0.039	0.048	0.053
4	0.006	0.022	0.027	0.03
5	0.004	0.014	0.017	0.019
6	0.003	0.01	0.012	0.013
7	0.002	0.007	0.009	0.01
8	0.0015	0.005	0.008	0.007
9	0.0012	0.004	0.005	0.006
10	0.001	0.0035	0.004	0.004

Table 5: Effect of β on the performance measures E (T)

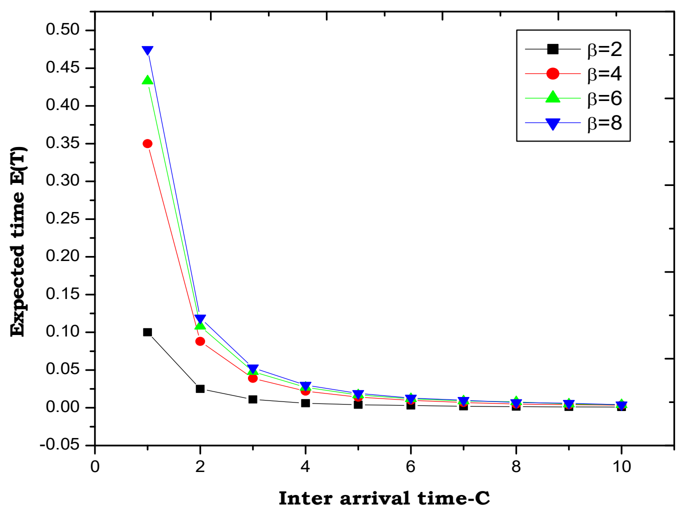

Figure 5 : The changes in E(T) due to changes in β

c	β=2	β=4	β=6	β=8
1	0.01	0.367	0.939	1.579
2	0.003	0.092	0.235	0.395
3	0.001	0.041	0.104	0.175
4	0.0006	0.023	0.059	0.099
5	0.0004	0.015	0.038	0.063
6	0.0003	0.01	0.026	0.044
7	0.0002	0.008	0.019	0.032
8	0.00025	0.007	0.015	0.025
9	0.00012	0.005	0.012	0.019
10	0.0001	0.004	0.009	0.016

Table 6: Effect of β on the performance measures E (T)

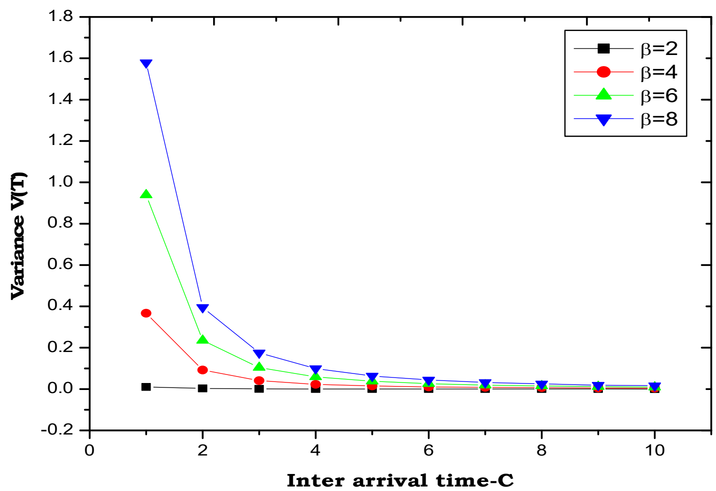

Figure 6 : The changes in V(T) due to changes in β

CONCLUSIONS

When μ is kept fixed the inter-arrival time $'c'$ which follows exponential distribution, is an increasing case by the process of time to recruitment. Therefore, the value of the expected time $E(T)$ to cross the time to recruitment is found to be decreasing, in all the cases of the parameter value $\mu = 0.5, 1, 1.5, 2$. When the value of the parameter μ increases, the expected time is also found decreasing, this is observed in Figure 1. The same case is found in Variance $V(T)$ which is observed in Figure 2.

When β is kept fixed and the inter-arrival time $'c'$ increases, the value of the expected time $E(T)$ to cross the time to recruitment is found to be decreasing, in all the cases of the parameter value $\beta = 0.5, 1, 1.5, 2$. When the value of the parameter β increases, the expected time is found increasing, this is indicated in Figure 3. The same case is observed in the antigenic diversity of seroconversion of Variance $V(T)$ which is observed in Figure 4.

When d is kept fixed and the inter-arrival time $'c'$ increases, the value of the expected time $E(T)$ to cross the time to recruitment is found to be decreasing, in all the cases of the parameter value $d = 0.5, 1, 1.5, 2$. When the value of the parameter d increases, the expected time is found increasing, this is indicated in Figure 5. The same case is observed in the antigenic diversity of seroconversion of Variance $V(T)$ which is observed in Figure 6.

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