A Mathematical Two Parameter Lindley Distribution on Continuous glucose monitoring in pregnant women with type 1 diabetes

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Abstract

Several lifetime data are investigated using statistical analysis based on a respective statistical distribution. However, many of the life time data are still needed an attention in favour of statistical view. In this paper, we have employed a Two Parameters Lindley Distribution to analyse patterns of continuous glucose monitoring (CGM) data for associations with large for gestational age (LGA) infants and an adverse neonatal composite outcome (NCO) in pregnancies in women with type 1 diabetes.

Key words: Lindley distribution, Parameter, diabetes,

1. INTRODUCTION

Lindley distribution (RD) is a continuous probability distribution for positive-valued random variables. It is a special case of the Lindley distribution and it is broadly used for survival analysis [4]. Lindley distribution [4,5] was introduced as one parameter distribution. to analyse the life data for the continuous glucose monitoring (CGM) data for associations with large for gestational age (LGA) infants and an adverse neonatal composite outcome (NCO) in pregnancies in women with type 1 diabetes.

Despite improved glycaemic control, the prevalence of macrosomia and large for gestational age (LGA) remains high in babies born to women with type 1 diabetes, affecting approximately one-half of these newborn infants [1,2].

In addition to an increased risk of obstetric and neonatal adverse outcomes, LGA infants have an increased risk of developing obesity, diabetes and cardiovascular disease in later life [8-9]. Continuous glucose monitoring (CGM) technology provides unique insights into daily glycaemic control and permits a better understanding of how glycaemic patterns and glucose variability may influence pregnancy outcomes [6].

Numerous lifetime data used in statistical analysis depends on a particular statistical distribution. Knowledge of suitable distribution of real data will extremely ameliorates the efficiency and the power of the statistical tests involved with it. Therefore, several distributions are suggested for modelling lifetime data. However, there are still many life time data that does not follow any distribution and hence there is a need to extend some new distributions. Here we suggested a new distribution for fitting lifetime data using one of the well known distribution function generation methods. The aim of the study was to determine patterns of maternal glucose control during different phases of pregnancy and to examine whether these patterns are associated with LGA and a predefined adverse neonatal composite outcome (NCO) using two parameters Lindley distribution.

2. METHODOLOGY

2.1. MATHEMATICAL MODEL

2.1.1. TWO PARAMETER LINDLEY DISTRIBUTION

A Two – parameter Lindley Distribution is one of the famous distribution [7] and of its probability density function defined as follows,

$$f(x;a,\theta) = \frac{\theta^2}{\theta + a}(1 + ax)e^{-\theta x}, \quad x > 0, \theta > 0, a > -\theta$$

The probability density function can be shown as a mixture of exponential and gamma (2, 0) distribution written of the form,

$$f(x;\alpha,\theta) = pf_1(x) + (1-p)f_2(x)$$

Where

$$p = \frac{\theta}{\theta = a}, f_{1\,(x)=\theta e^{-\theta x}}$$

And

$$f_2(x) = \theta^2 x e^{-\theta x}$$

And so

$$f'(x) = 0 \; gives \; x = \frac{a-\theta}{a\theta}$$

From this, it follows that

1) For $\theta < a, x_0 = \frac{a-\theta}{a\theta}$ is the unique critical point at which $f(x)$ is maximum

   For $\theta \geq a, f'(x) \leq 0 \; i.e. f(x)$ is decreasing in x. Therefore, the mode of the distribution is given by

$$\text{Mode} = \begin{cases} \frac{a-\theta}{a\theta}, \theta < a \\ 0, \end{cases}$$

The cumulative distribution function (c. d. f) of the two – parameter LD is given by

$$\text{F}(x) = 1 - \frac{\theta+a+a\theta x}{\theta+a} e^{-\theta x}$$

$$x > 0, 0, a > -\theta$$

B. Moments and related measures

The r th moment about origin of the two - parameter LD has been obtained as

$$u_r' = \frac{\Gamma(r+1)(\theta+a+ar)}{\theta^r(\theta+a)} \; ; r = 1,2, \ldots$$

Taking $r = 1,2,3$ and 4 is the first four moments about origin are obtained as

$$u_1' = \frac{\theta+2a}{\theta(\theta+a)}, u_2' = \frac{2(\theta+3a)}{\theta^2(\theta+a)},$$

$$u_3' = \frac{6(\theta+4a)}{\theta^3(\theta+a)}, u_4' = \frac{24(\theta+5a)}{\theta^4(\theta+a)},$$

It can be easily verified that for $a = 1$, the central moment of the two – parameter LD reduce to the respective moments of the one parameter LD.

The coefficients of variation (γ), skewness $(\sqrt{\beta_1})$ and the kurtosis (β_2) of the two – parameter LD are given by

$$\gamma = \frac{\sigma}{u_1'} = \frac{\sqrt{\theta^2 4\theta a + 2a^2}}{\theta + 2a}$$

$$\sqrt{\beta_1} = \frac{u_3}{u_2^{3/2}} = \frac{2(\theta^3 + 6\theta^2 a + 6\theta a^2 + 2a^3)}{(\theta^2 + 4\theta a + 2a^2)^{3/2}}$$

$$u_4 = \frac{3(3\theta^4 + 24\theta^3 a + 24\theta^2 a + 20\theta^2 a^2 - 88\theta a^2 + 120\theta a^2 + 8a^4)}{\theta^4(\theta + a)^4}$$

$$\beta_2 = \frac{u_4}{u_2^2} = \frac{3(3\theta^4 + 24\theta^3 a + 24\theta^2 a + 20\theta^2 a^2 - 88\theta a^2 + 120\theta a^2 + 8a^4)}{(\theta^2 + 4a\theta + 2a^2)^2}$$

C. **Hazard Rate Function**

For a continuous distribution with p.d.f. *f* (x) and c.d.f. F(x), the failure rate function is hazard rate function defined as

$$h(x) = lim \frac{P(X < x + \Delta x | X > x)}{\Delta x} = \frac{f(x)}{1 - F(x)}$$

The corresponding failure rate function, $h(x)$ of two – parameter Lindley distribution [9] written as follows,

$$h(x) = \frac{\theta^2(1 + ax)}{\theta + a + \theta ax)}$$

3. RESULTS

3.1. APPLICATION

3.1.1. Participants:

Kristensen et al.[3] performed a retrospective analysis of CGM data in women with type 1 diabetes who received pregnancy care between 2014 and 2017 at two large tertiary care antenatal clinics in Sweden (Skåne University Hospital and Östra/Sahlgrenska University Hospital). All women above 18 years of age using a CGM device compatible with the internet-based Diasend system (Glooko, Gothenburg, Sweden) were eligible for inclusion in the study. CGM data were available from 192 women. Of these, three women decided to opt out. Another three women were excluded because of: termination of pregnancy due to

chromosome aberration ($n = 1$); intrauterine fetal demise ($n = 1$); and multiple gestation ($n = 1$). After exclusion of these pregnancies, CGM data from 186 singleton pregnancies were available for analysis.

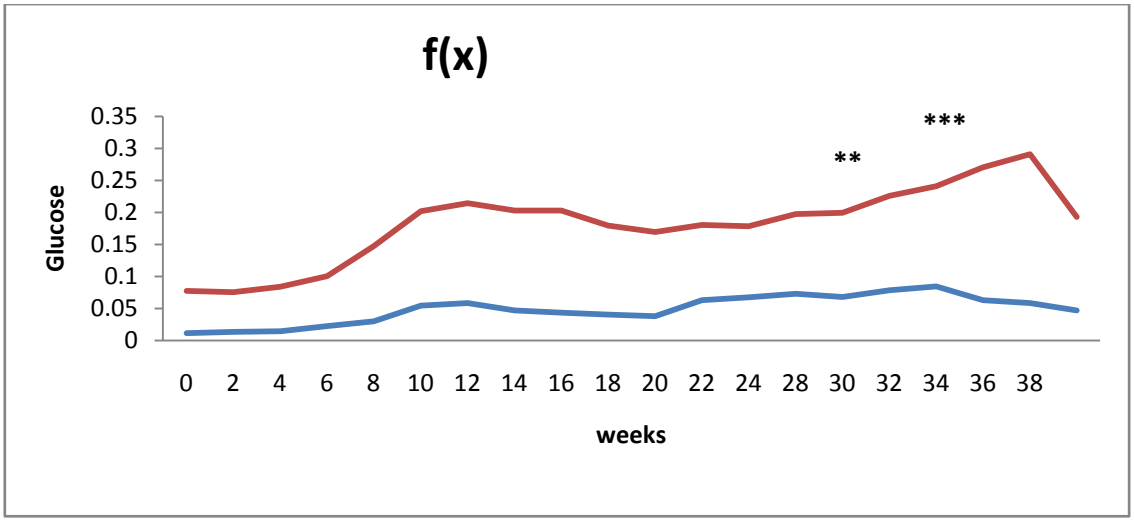
In the present study, we sought to gain local experience of wearing CGM during pregnancy. Despite the use of CGM throughout pregnancy, the day-to-day glucose control was not optimal and the incidence of LGA remained high. There is a need for greater support from the diabetes team during pregnancy for technical assistance and intensified focus on postprandial hyperglycaemia, including dietary advice/carbohydrate counting and a supported active approach to prandial insulin adjustments. Because of ease of use and low cost, the iCGM system has become increasingly popular in Sweden among both individuals with diabetes and caregivers. The system has been considered safe and accurate for use in pregnant women with diabetes [8]. It is our clinical experience that many women prefer to use iCGM rather than rtCGM in pregnancy. Further randomised trials to assess the impact of iCGMvs rtCGM on glucose control and neonatal outcomes in pregnancy are warranted..

4. MATHEMATICAL RESULTS

4.1. The probability density function of two parameters Lindley distribution.

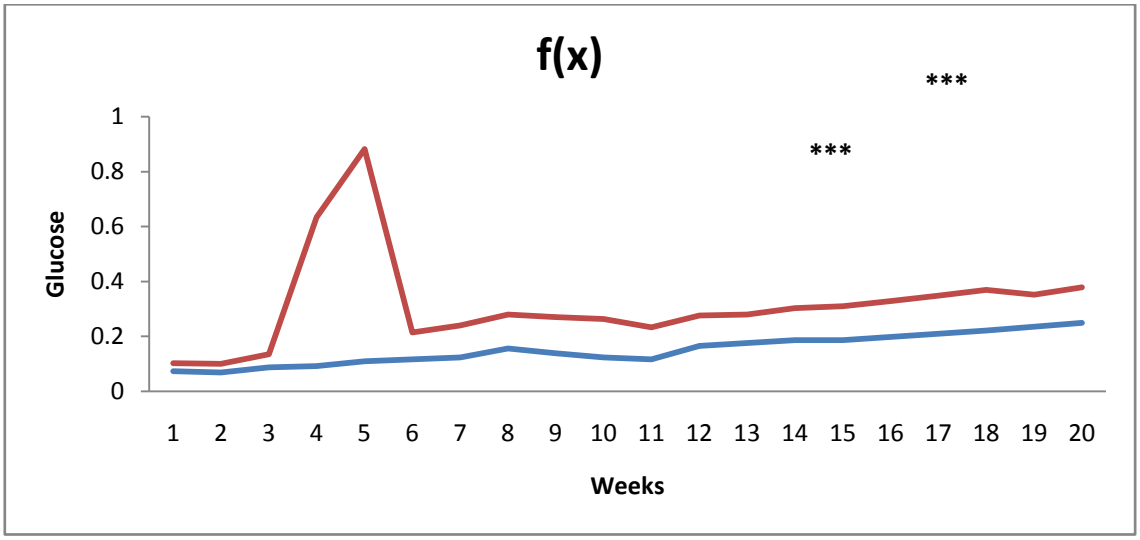
Probability density functions $f(x)$ of two parameters Lindley distribution analysis on graphical levels of interstitial glucose levels during pregnancy in women with (white) and without (black) fetal overgrowth (LGA) and interstitial glucose levels during pregnancy in women with (white) and without (black) short-term neonatal complications

is shown in mathematical figure 1(a-b). In parallel to Kristensen et al [3] the two parameter LD $f(x)$ plot shows interstitial glucose levels during pregnancy in women with (white) and without (black) fetal overgrowth (LGA) (figure 1b). Similar results were observed with interstitial glucose levels during pregnancy in women with (white) and without (black) short-term neonatal complications Glucose levels too (figure 1a).



Mathematical Figure: 1a

The two parameters lindley distribution of its probability density function of graphical levels of interstitial glucose levels during pregnancy in women with (white) and without (black) fetal overgrowth (LGA).



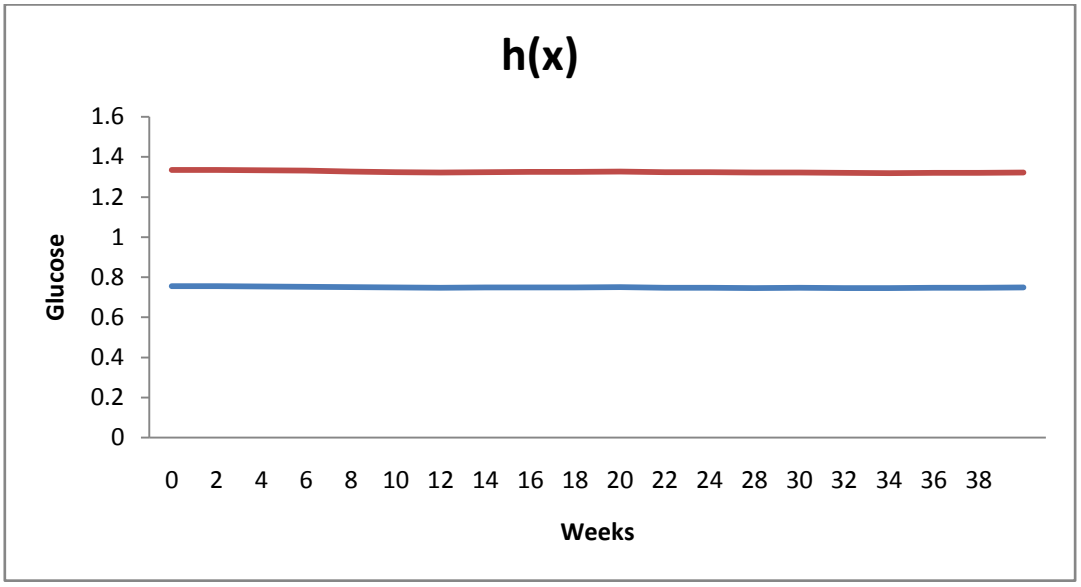
Mathematical Figure: 1b

The two parameters lindley distribution of its probability density function of graphical levels of interstitial glucose levels during pregnancy in women with (white) and without (black) short-term neonatal complications

4.2. The Hazard function of the two parameters Lindley distribution.

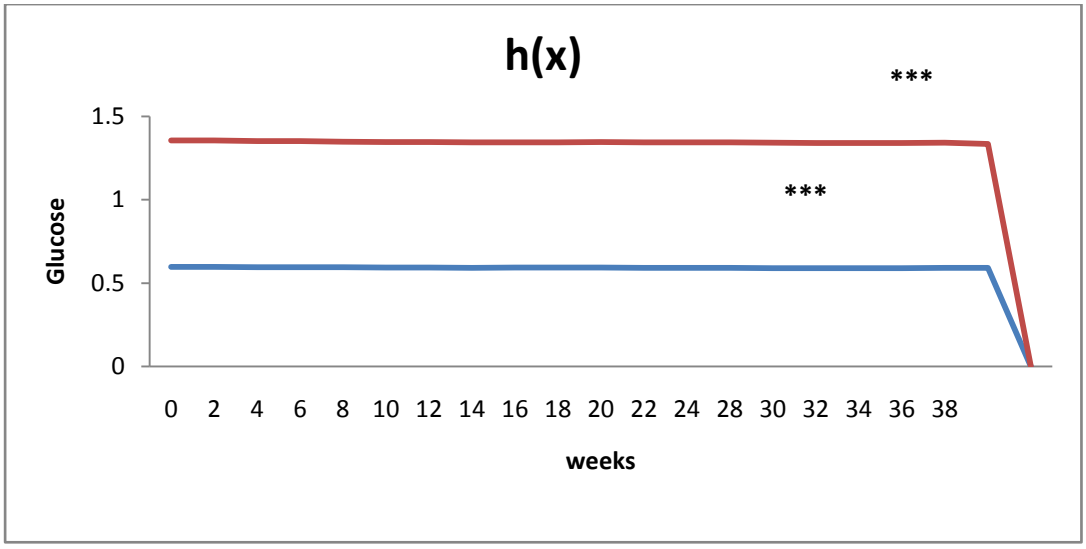
Hazard functions $f(x)$ of two parameters Lindley distribution analysis on graphical levels of interstitial glucose levels during pregnancy in women with (white) and without (black) fetal overgrowth (LGA) and interstitial glucose levels during pregnancy in women with (white) and without (black) short-term neonatal complications

is shown in mathematical figure 2(a-b). In parallel to Kristensen et al.[3] the two parameter LD $f(x)$ plot shows decreased levels of interstitial glucose levels during pregnancy in women with (white) and without (black) fetal overgrowth (LGA) (figure 2b). the results were observed with interstitial glucose levels during pregnancy in women with (white) and without (black) short-term neonatal complications Glucose levels too (figure 2a).



Mathematical Figure: 2a

The two parameters lindley distribution of its Hazard function of graphical levels of interstitial glucose levels during pregnancy in women with (white) and without (black) fetal overgrowth (LGA



Mathematical Figure: 2b

The two parameters lindley distribution of its Hazard function of graphical levels of of interstitial glucose levels during pregnancy in women with (white) and without (black) short-term neonatal complications

Conclusion

Probability density functions $f(x)$ of two parameters Lindley distribution analysis on graphical levels of interstitial glucose levels during pregnancy in women with (white) and without (black) fetal overgrowth (LGA) and interstitial glucose levels during pregnancy in women with (white) and without (black) short-term neonatal complications

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