ODD EDGE LABELING AND ODD ELEGANT LABELING IN LADDER AND DUPLICATE GRAPH OF LADDER GRAPH

Submitted in partial fulfilment of the requirements for the award of the degree of

Master of Science
in
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Submitted by

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Certificate

This is to certify that this is a bonafide record of the project presented by the students whose names are given below during Fourth semester in partial fulfilment of the requirements for the award of the degree of Master of science in Mathematics.

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Date:

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(B. Priyanka Patro)

(Kanhu Charan Satapathy)

Contents

ABSTARCT

Alex Rosa has introduced the concept of graph labeling. The concept of odd edge labeling was introduced by V.Lakshmi alias Gomathi , A. Nagarjan and A. Nellai Murugan. The concept of odd elegant labeling was introduced by Chang. G. J., Hsu. D.f. and Rogers have introduced elegant labeling and Xiangqian Zhou, Bing Yao and Xiang'en chen and they have proved that every lobster is odd elegant. They have proved the existence of the same in certain graphs. Thulukkanam has proved the existence of Odd Edge labeling in duplicate graphs of path and star related graphs.

Motivated by the above studies we will prove the existance of odd edge labeling and odd elegant labeling in Ladder graph and Duplicate graph of ladder graph.

Chapter 1

INTRODUCTION

The history of graph theory may be specifically traced to 1735, when the Swiss mathematician Leonhard Euler solved the Konigsberg bridge problem. Graph theory is a branch of mathematics concerned with networks of points connected by lines. It is a mathematical structure that contains non empty set of vertices and set of edges. The concept of graph labeling was introduced by Rosa in 1967.

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. if the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or edge labeling).

1.1 Preliminaries

In this section, we give the basic notions relevant to this paper.

Definition 1.1.1. *A graph is a mathematical structure consisting of two sets V and E where V is the non-empty set of vertices and E is the set of edges.*

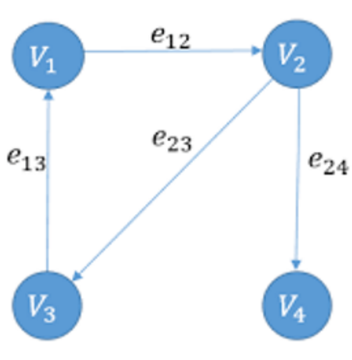

Figure 1.1:

Definition 1.1.2. *A graph which does not contain any self loop and multiple edges is called as simple graph.*

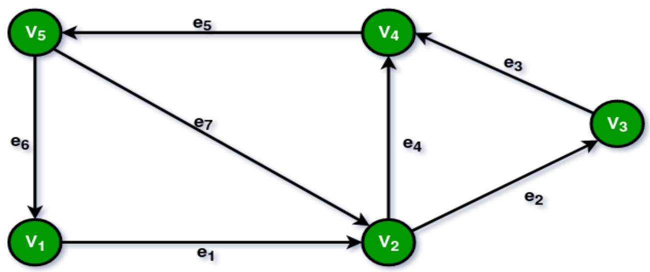

Figure 1.2:

Definition 1.1.3. *In a graph, if the vertex set* V *can be expressed as two disjoint sets say* X *and* Y *such that one end vertex of each edge lies in* X *, whereas other end vertex lies in* Y *, then that graph is called as bipartite graph .*

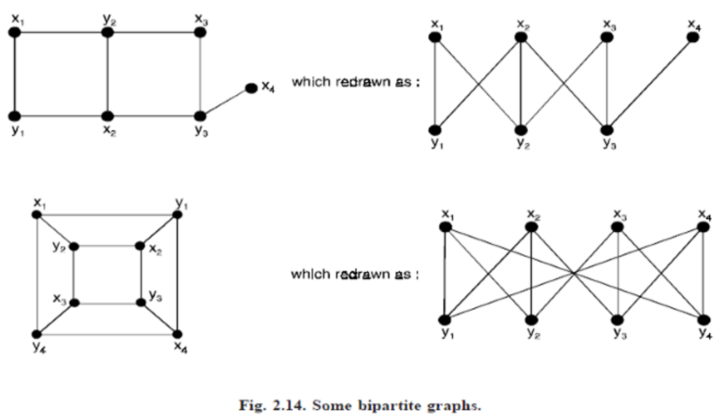

Figure 1.3:

Definition 1.1.4. *A simple graph of n vertices* ($n \geq 3$) *and n edges forming a cycle of length n is called as a cycle graph. In a cycle graph, all the vertices are of degree 2.*

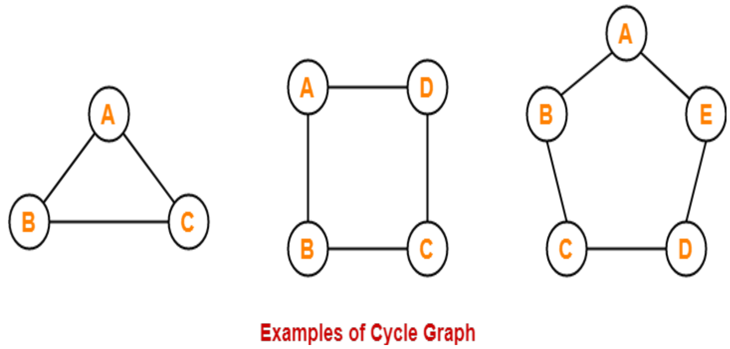

Figure 1.4:

Definition 1.1.5. *A graph in which degree of all the vertices is same is called as a regular graph. If all the vertices in a graph are of degree 'k', then it is called as a "k-regular graph".*

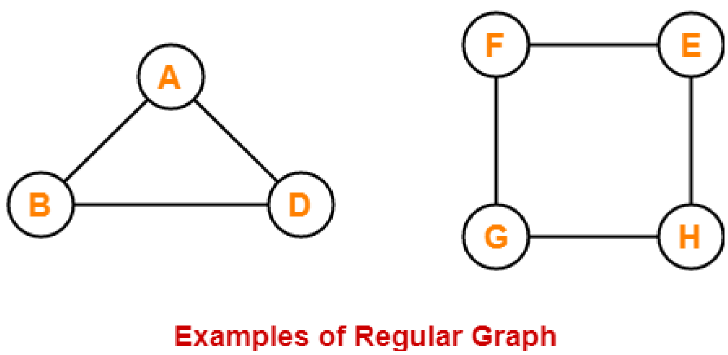

Figure 1.5:

Definition 1.1.6. *Euler Graph is a connected graph in which all the vertices are of even degree.*

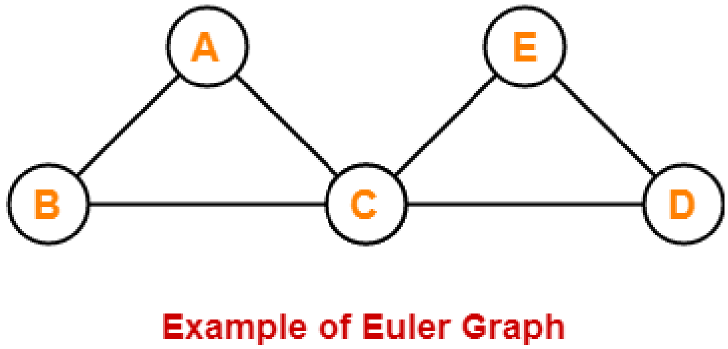

Figure 1.6:

Definition 1.1.7. *The ladder graph L_m is a planar undirected graph with $2m$ vertices and $3m-2$ edges .It is obtained as the Cartesian product of two path graphs, one of which has only one edge $L_{m,1} = P_m \times P_1$, where m is the number of rungs in the ladder.*

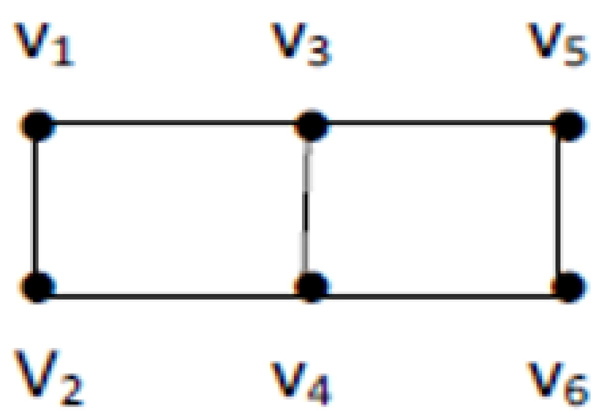

Figure 1.7:

Definition 1.1.8. *Let $G(V, E)$ be a simple graph. A Duplicate graph of G is $DG = (V_1, E_1)$ where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \emptyset$ and $f : V \to V'$ is bijective and the edges set E_1 of DG is defined as : the edge ab in E if and only if both ab' and $a'b$ are edges in E_1.*

The construction of duplicate graph of ladder graph is given below.

Case 1: *For* $K = 2, 4, 6, ..., (2m-2)$

$$v_k v'_{k+2} \leftarrow e_{\frac{3k}{2}}; v_{k+2} v'_k \leftarrow e'_{\frac{3k}{2}}$$

Case 2: *For* $K = 1, 2, 3, ..., m$

$$v_{2k-1} v'_{2k} \leftarrow e_{3k-2}; v_{2k} v'_{2k-1} \leftarrow e'_{3k-2}$$

Case 3: *For* $K = 1, 2, 3, ..., (m-1)$

$$v_{2k-1} v'_{2k+1} \leftarrow e_{3k-1}; v_{2k+1} v'_{2k-1} \leftarrow e'_{3k-1}$$

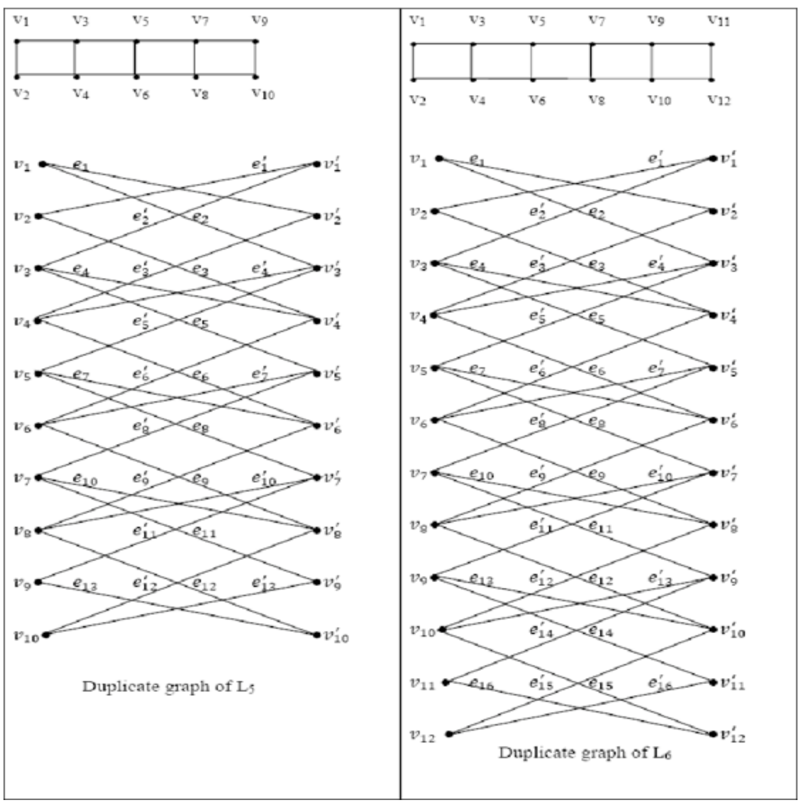

Figure 1.8: Duplicate graph of L_5 and L_6

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Chapter 2

ODD EDGE LABELING

The concept of odd edge labeling was introduced by V. Lakshmi alias Gomathi, A. Nagarajan and A. Nellai Murugan. They have proved the existence of odd edge labeling in certain graphs.

Motivated by them, we will prove that the path graph, star graph, ladder graph, duplicate ladder graph admits odd edge labeling.

2.0.1 Definition

Let G be a (p, q) graph. A one-one function $\Phi : V(G) \to \{0, 1, 2, 3, ..., q\}$ is said to be an odd edge labeling of G, if for every edge $e = ab \in E(G), \Phi(a) + \Phi(b)$ is odd and $\Phi^{\star}(E(G)) = \{1, 3, 5, ..., 2q - 1\}$.

2.0.2 Theorem

Theorem 2.0.1. *The path graph $P_m, m \geq 2$, admits odd edge labeling.*

Proof. The $2m + 2$ vertices of the path graph are labeled using $0, 1, 2, ..., q$
Case (1): When $m \equiv 1(mod2)$
The induced function $\Phi^{\star}$ is defined by $\Phi^{\star}(uv) = \Phi(u) + \Phi(v)$ assigns the labels to the m edges as follows.

The m edges namely $e_1, e_2^{'}, e_3, e_4^{'}, e_5, e_6^{'}, e_7, e_8^{'}, ..., e_{m-3}^{'}, e_{m-2}, e_{m-1}^{'}, e_m$ receives the labels $1, 3, 5, 7, 9, 11, 13, 15, ..., 2m-7, 2m-5, 2m-3, 2m-1$ respectively and the remaining m edges namely $e_1^{'}, e_2, e_3^{'}, e_4, e_5^{'}, e_6, e_7^{'}, e_8, ..., e_{m-3}, e_{m-2}^{'}, e_{m-1}, e_m^{'}$ receives the labels $4m + 1, 4m - 1, 4m - 3, 4m - 7, 4m - 9, 4m - 11, 4m - 13, 4m - 15, ..., 2m + 9, 2m + 7, 2m + 5, 2m + 3$

respectively and the edge e_{m+1} is labeled with $2m+1$.

Case (2): When $m \equiv 0(mod2)$
The induced function $\Phi^{\star}$ is defined by $\Phi^{\star}(uv) = \Phi(u) + \Phi(v)$ assigns the labels to the m edges as follows.

The m edges namely $e_1, e_2^{'}, e_3, e_4^{'}, e_5, e_6^{'}, ..., e_{m-3}, e_{m-2}^{'}, e_{m-1}, e_m^{'}$ receives the labels $1, 3, 5, 7, 9, 11, 13, 15, 17, ..., 2m-7, 2m-5, 2m-3, 2m-1$ respectively and the remaining m edges namely $e_1^{'}, e_2, e_3^{'}, e_4, e_5^{'}, e_6, ..., e_{m-3}^{'}, e_{m-2}, e_{m-1}^{'}, e_m$ receives the labels $4m+1, 4m-1, 4m-3, 4m-7, 4m-9, ..., 2m+9, 2m+7, 2m+5, 2m+3$ respectively and the edge e_{m+1} is labeled with $2m+1$.
Thus the q edges are labeled with $1, 3, 5, 7, ..., 4m+1$. Therefore the extended duplicate graph of path graph $P_m, m \geq 2$ admits odd edge labeling. □

2.0.3 Algorithm-OELDPG

$V \leftarrow \left\{v_1, v_2, ..., v_{m+1}, v_1^{'}, v_2^{'}, ..., v_{m+1}^{'}\right\}$
$E \leftarrow \left\{e_1, e_2, ..., e_{m+1}, e_1^{'}, e_2^{'}, ..., e_m^{'}\right\}$

Case (1): When $m \equiv 1(mod2)$
For $1 \leq k \leq \frac{m+1}{2}$
$v_{2k-1} \leftarrow 2k-1$
$v_{2k} \leftarrow 2m-2k+2$
$v_{2k-1}^{'} \leftarrow 2m-2k+3$
$v_{2k}^{'} \leftarrow 2k-1$

Case (2): When $m \equiv 0(mod2)$
For $1 \leq k \leq \frac{m}{2}$
$v_{2k-1} \leftarrow 2k-2$
$v_{2k} \leftarrow 2m-2k+2$
$v_{2k-1}^{'} \leftarrow 2m-2k+3$
$v_{2k}^{'} \leftarrow 2k-1$

For $k = m+1$
$v_k \leftarrow m$
$v_k^{'} \leftarrow m+1$

2.0.4 Illustration

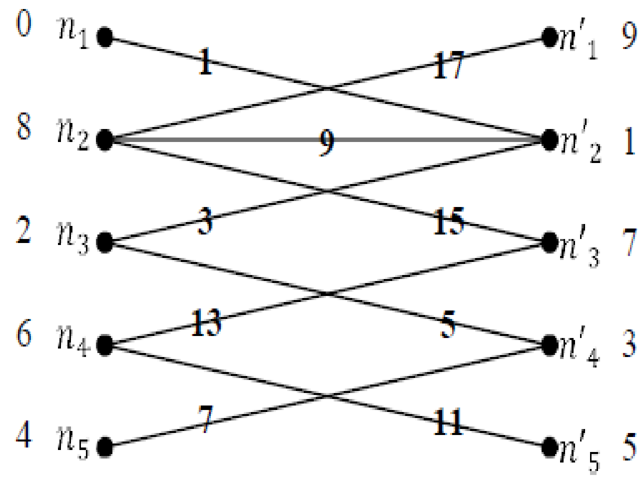

Figure 2.1: Odd Edge Labeling in P_4

2.0.5 Theorem

Theorem 2.0.2. *The star graph $S_m, m \geq 2$, admits odd edge labeling.*

Proof. The $2m$ vertices are labeled with $0, 1, 2, ..., q$. The induced function $\Phi^{\star}$ defined by $\Phi^{\star}(ab) = \Phi(a)+\Phi(b)$, assigns to the $m-1$ edges $e_1, e_2, ..., e_{m-1}$ the labels $1, 3, 5, 7, ..., 2m-3$, to the edges e_m the label $2m-1$ and to the $m-1$ edges $e_1^{'}, e_2^{'}, e_3^{'}, ..., e_{m-1}^{'}$ the labels $2m+1, 2m+3, 2m+5, ..., 4m-3$ respectively.

Thus the $2m-2(=q)$ edges are labeled with $1, 3, 5, 7, ..., 4m-3$.

Therefore the graph $S_m, m \geq 2$ admits odd edge labeling.

2.0.6 Algorithm-OELSG

$V \leftarrow \left\{v_1, v_2, ..., v_m, v_1^{'}, v_2^{'}, ..., v_m^{'}\right\}$
$E \leftarrow \left\{e_1, e_2, ..., e_m, e_1^{'}, e_2^{'}, ..., e_{m-1}^{'}\right\}$

Fix $v_1^{'} \leftarrow 2m-1$

For $1 \leq k \leq m$
$v_k \leftarrow 2k - 2$

For $1 \leq k \leq m - 1$
$v_{k+1} \leftarrow 2k - 1$

2.0.7 Illustration

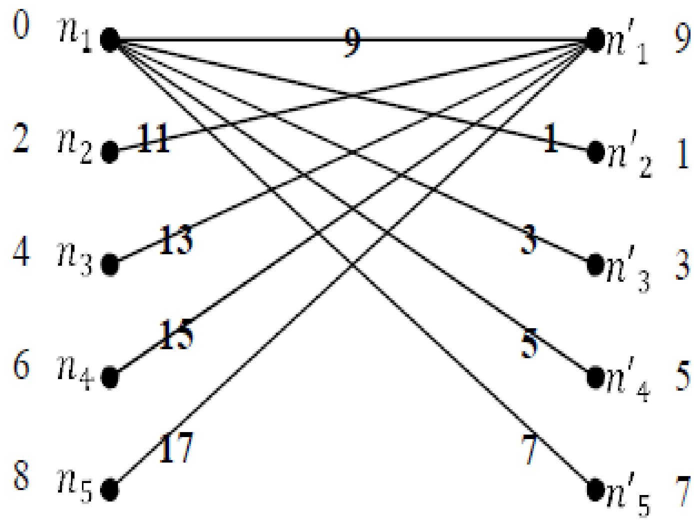

Figure 2.2: Odd Edge Labeling in S_5

□

2.0.8 Theorem

Theorem 2.0.3. *The ladder graph $L_m, m \geq 2$, admits odd edge labeling.*

Proof. The m vertices of the ladder graph namely $v_1, v_3, v_5, v_7, ...v_{2m-1}$ are labeled with $0, 3, 6, 9, ...3m - 3$ respectively and the remaining m vertices namely $v_2, v_4, v_6, v_8, ...v_{2m}$ are labeled with $1, 4, 7, 10, ...3m - 2$ respectively.
Using the induced function $\Phi^\star$ defined by $\Phi^\star(uv) = \Phi(u) + \Phi(v)$ the edges are labeled as below.
$\Phi^\star(v_{2i-1}v_{2i+1}) = 6i - 3; (1 \leq i \leq m - 1)$
$\Phi^\star(v_{2i}v_{2i+2}) = 6i - 1; (1 \leq i \leq m - 1)$
$\Phi^\star(v_{2i-1}v_{2i}) = 6i - 5; (1 \leq i \leq m)$

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Thus, the $3m - 2$ edges are labeled with $1, 3, 5, 2q - 1$.

Hence, the ladder graph $L_m, m \geq 2$ admits odd edge labeling.

□

2.0.9 Illustration

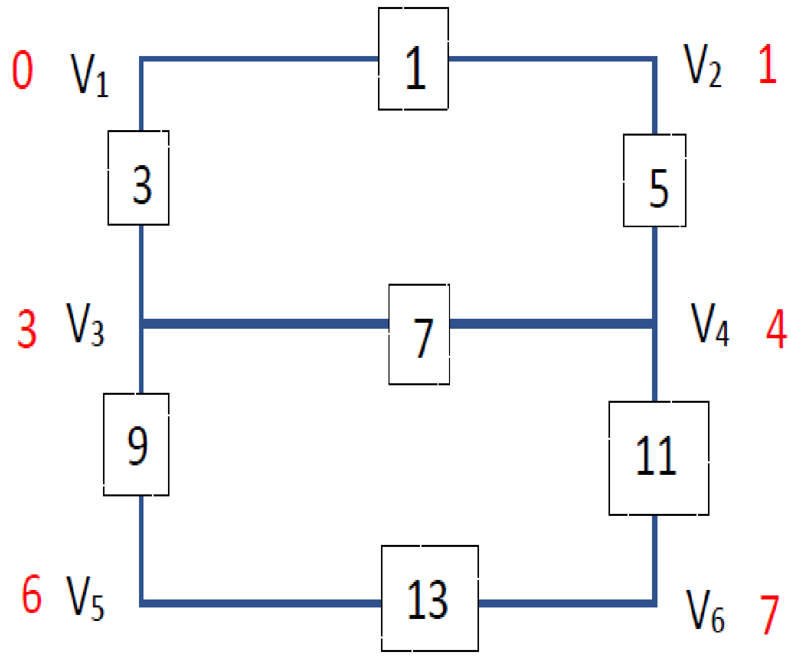

Figure 2.3: Odd Edge Labeling in L_3

2.0.10 Theorem

Theorem 2.0.4. *For $m \geq 2$, the duplicate graph of ladder graph admits odd edge labeling.*

Proof. Using the algorithm, the $4m$ vertices of the duplicate graph of ladder graph $DG(L_m)$ are labeled using $\{0, 1, 2, ...q\}$.

Case(1): When m is odd.
The induced function $\Phi^{\star}$ is defined by $\Phi^{\star}(uv) = \Phi(u) + \Phi(v)$ assigns the labels to the edges as follows.

The $3m-2$ edges namely $e_1, e_2, e_3^{'}, e_4^{'}, e_5^{'}, e_6, e_7, e_8, e_9^{'}, e_{10}^{'}, e_{11}^{'}, e_{12}, e_{13}, e_{14}, ..., e_{3m-9}, e_{3m-8}, e_{3m-7}, e_{3m-6}^{'}, e_{3m-5}^{'}, e_{3m-4}^{'}, e_{3m-3}, e_{3m-2}$ receives the labels $1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, ..., q-15, q-13, q-11, q-9, q-7, q-5, q-3, q-1$ respectively and the remaining $3m-2$ edges namely $e_1^{'}, e_2^{'}, e_3, e_4, e_5, e_6^{'}, e_7^{'}, e_8^{'}, e_9, e_{10}, e_{11}, e_{12}^{'}, e_{13}^{'}, e_{14}^{'}, ..., e_{3m-9}^{'}, e_{3m-8}^{'}, e_{3m-7}^{'}, e_{3m-6}, e_{3m-5}, e_{3m-4}, e_{3m-3}^{'}, e_{3m-2}^{'}$ receives the labels $q+1, q+3, q+5, q+7, q+9, q+11, ..., 2q-9, 2q-7, 2q-5, 2q-3, 2q-1$ respectively.

Hence the $6m-4$ edges are labeled with $1, 3, 5, ..., 2q-1$.

Case(2): When m is even.
The induced function $\Phi^{\star}$ is defined by $\Phi^{\star}(uv) = \Phi(u) + \Phi(v)$ assigns the labels to the edges as follows.

The $3m-2$ edges namely $e_1, e_2, e_3^{'}, e_4^{'}, e_5^{'}, e_6, e_7, e_8, e_9^{'}, e_{10}^{'}, e_{11}^{'}, e_{12}, e_{13}, e_{14}, ..., e_{3m-12}, e_{3m-11}, e_{3m-10}, e_{3m-9}^{'}, e_{3m-8}^{'}, e_{3m-7}^{'}, e_{3m-6}, e_{3m-5}, e_{3m-4}, e_{3m-3}^{'}, e_{3m-2}^{'}$ receives the labels $1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, ..., q-21, q-19, q-17, q-15, q-13, q-11, q-9, q-7, q-5, q-3, q-1$ respectively and the remaining $3m-2$ edges namely $e_1^{'}, e_2^{'}, e_3, e_4, e_5, e_6^{'}, e_7^{'}, e_8^{'}, e_9, e_{10}, e_{11}, e_{12}^{'}, e_{13}^{'}, e_{14}^{'}, ..., e_{3m-12}^{'}, e_{3m-11}^{'}, e_{3m-10}^{'}, e_{3m-9}, e_{3m-8}, e_{3m-7}, e_{3m-6}^{'}, e_{3m-5}^{'}, e_{3m-4}^{'}, e_{3m-3}, e_{3m-2}$ receives the labels $q+1, q+3, q+5, q+7, q+9, q+11, ..., 2q-21, 2q-19, 2q-17, 2q-15, 2q-13, 2q-11, 2q-9, 2q-7, 2q-5, 2q-3, 2q-1$ respectively.

Hence the $6m-4$ edges are labeled with $1, 3, 5, ..., 2q-1$. □

2.0.11 Algorithm-OELDLG

$V \leftarrow \left\{v_1, v_2, v_3, ..., v_{2m}, v_1^{'}, v_2^{'}, v_3^{'}, ..., v_{2m}^{'}\right\}$
$E \leftarrow \left\{e_1, e_2, e_3, ..., e_{3m-2}, e_1^{'}, e_2^{'}, e_3^{'}, ..., e_{3m-2}^{'}\right\}$

Case(1): When m is odd.

Fix $v_1 \leftarrow 1; v_1^{'} \leftarrow (\frac{q}{2}) + 1$

For $1 \leq k \leq \frac{m-1}{2}$

$v_k \leftarrow 6k-1$
$v_{4k+1} \leftarrow 6k+1$
$v_{4k-2} \leftarrow \frac{q}{2} + (6k-6)$
$v_{4k-1} \leftarrow \frac{q}{2} + (6k-4)$

$v_{4k-2}^{'} \leftarrow 6k-6$
$v_{4k-1}^{'} \leftarrow 6k-4$

$v'_{4k} \leftarrow \frac{q}{2} + (6k-1)$
$v'_{4k+1} \leftarrow \frac{q}{2} + (6k+1)$

For $k = 2m$
$v_k \leftarrow q-1$
$v'_k \leftarrow \frac{q}{2} - 1$

Case(2): When m is even.

Fix $v_1 \leftarrow 1; v'_1 \leftarrow (\frac{q}{2}) - 1$

For $1 \leq k \leq \frac{m}{2} - 1$

$v_k \leftarrow 6k-1$
$v_{4k+1} \leftarrow 6k+1$
$v'_{4k} \leftarrow \frac{q}{2} + (6k-3)$
$v'_{4k+1} \leftarrow \frac{q}{2} + (6k-1)$

For $1 \leq k \leq \frac{m}{2}$

$v_{4k-2} \leftarrow \frac{q}{2} + (6k-4)$
$v_{4k-1} \leftarrow \frac{q}{2} + (6k-2)$
$v'_{4k-2} \leftarrow (6k-6)$
$v'_{4k-1} \leftarrow (6k-4)$

For $k = 2m$
$$v_k \leftarrow \frac{q}{2} + 1$$
$v'_k \leftarrow q-1$

2.0.12 Illustration

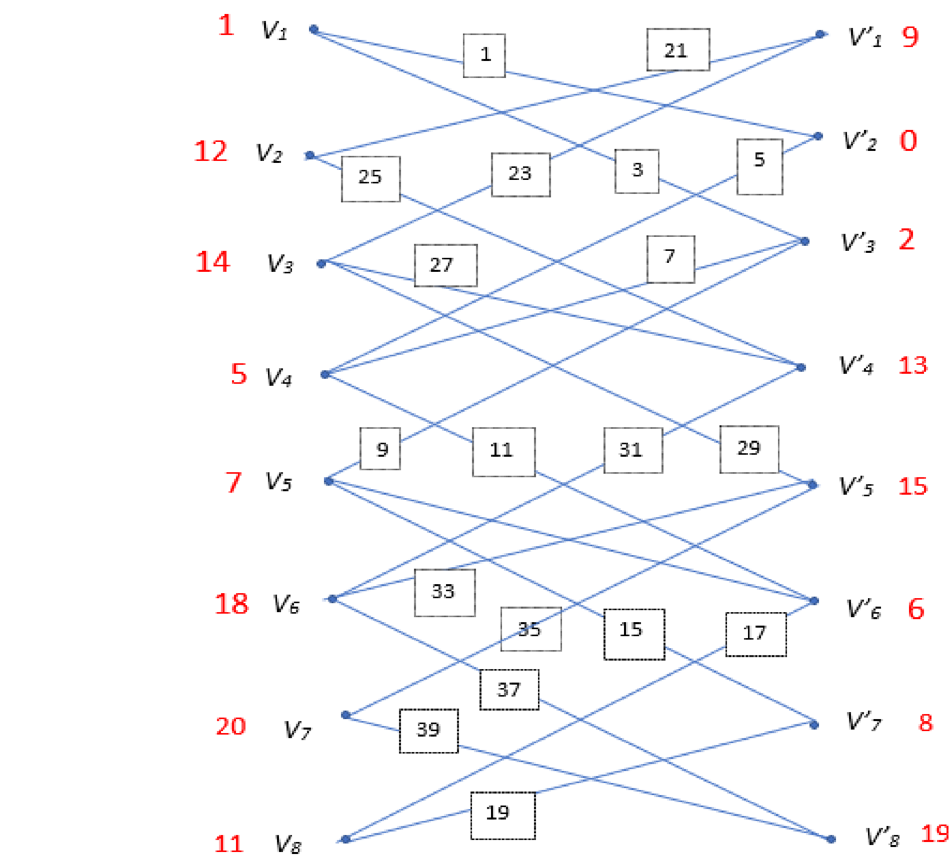

Figure 2.4: Odd Edge Labeling in DL_4

Chapter 3

ODD ELEGANT LABELING

The concept of odd elegant labeling was introduced by Chang. G. J. , Hsu. D.f. and Rogers have introduced elegant labeling. The concept of odd elegant labeling was introduced by Xiangqian Zhou, Bing Yao and Xiang'en chen and they have proved that every lobster is odd elegant.

Motivated by them, we will prove that the path graph, star graph, ladder graph, duplicate ladder graph admits odd elegant labeling.

3.0.1 Definition

A graph G with q edges is odd elegant if it admits a mapping $\Phi : V \rightarrow 0, 1, 2, ..., 2q-1$ with $\Phi(a) \neq \Phi(b)$, for distinct $a, b \epsilon V(G)$ and the label $\Phi^{\star}(ab)$ of every edge $ab \epsilon E(G)$ is defined as $\Phi^{\star}(ab) = [\Phi(a) + \Phi(b)](mod 2q)$ such that the set of edge labels is equal to $1, 3, 5, ..., 2q-1$.

3.0.2 Theorem

Theorem 3.0.1. *The path graph $P_m, m \geq 2$, admits odd elegant labeling.*

Proof. The $2m+2$ vertices of the path graph are labeled using $0, 1, 2, ..., 2q-1$.

Case (1): When $m \equiv 1(mod 2)$
Using induced function $\Phi^{\star}(ab) = [\Phi(a)+\Phi(b)](mod 2q)$, the $\frac{m+1}{2}$ edges $e_1, e_3, e_5, ..., e_{m-2}, e_m$ receives the labels $1, 3, 5, 7, ...2m-5, 2m-1$ respectively, the $\frac{m-1}{2}$ edges $e_2, e_4, e_6, ..., e_{m-3}, e_{m-1}$ receives the labels $4m-3, 4m-5, 4m-7, ..., 2m+7, 2m+3$ respectively, the $\frac{m+1}{2}$ edges $e'_1, e'_3, e'_5, ..., e'_{m-2}, e'_m$ receives the labels $4m-1, 4m-5, 4m-9, ..., 2m+5, 2m+1$ respec-

tively, the $\frac{m-1}{2}$ edges $e_2^{'}e_4^{'}, e_6^{'}, ..., e_{m-3}^{'}, e_{m-1}^{'}$ receives the labels $3, 7, 11, ..., 2m-7, 2m-3$ respectively and the edges e_{m+1} receives label $4m+1$.

Case (2): When $m \equiv 0(mod2)$
Using induced function $\Phi^{\star}(ab) = [\Phi(a)+\Phi(b)](mod2q)$, the $\frac{m}{2}$ edges $e_1, e_3, e_5, ..., e_{m-3}, e_{m-1}$ receives the labels $1, 5, 9, ..., 2m-7, 2m-3$ respectively, the $\frac{m}{2}$ edges $e_2, e_4, e_6, ..., e_{m-2}, e_m$ receives the labels $4m-3, 4m-7, 4m-11, ..., 2m+5, 2m+1$ respectively,the $\frac{m}{2}$ edges $e_2^{'}, e_4^{'}, e_6^{'}, ..., e_{m-2}^{'}, e_m^{'}$ receives the labels $3, 7, 11, ..., 2m-5, 2m+1$ respectively and the edges e_{m+1} receives label $4m+1$.

Thus the $2m+1$ edges are labeled with $1, 3, 5, ..., 4m+1$.

Hence the extended duplicate graph of path graph $P_m, m \geq 2$ admits odd elegant labeling.

□

3.0.3 Algorithm-OELPG

$V \leftarrow \left\{v_1, v_2, ..., v_{m+1}, v_1^{'}, v_2^{'}, ..., v_{m+1}^{'}\right\}$
$E \leftarrow \left\{e_1, e_2, ..., e_{m+1}, e_1^{'}, e_2^{'}, ..., e_m^{'}\right\}$

Case (1): When $m \equiv 1(mod2)$
For $1 \leq k \leq \frac{m+1}{2}$

$v_{2k-1} \leftarrow 2k-2, v_{2k-1}^{'} \leftarrow 4m-2k+3$
$v_{2k} \leftarrow 4m-2k+2, v_{2k}^{'} \leftarrow 2k-1$

Case (2): When $m \equiv 0(mod2)$
For $1 \leq k \leq \frac{m}{2}+1$
$v_{2k-1} \leftarrow 2k-2, v_{2k-1}^{'} \leftarrow 4m-2k+3$

For $1 \leq k \leq \frac{m}{2}$
$v_{2k} \leftarrow 4m-2k+2, v_{2k}^{'} \leftarrow 2k-1$

3.0.4 Illustration

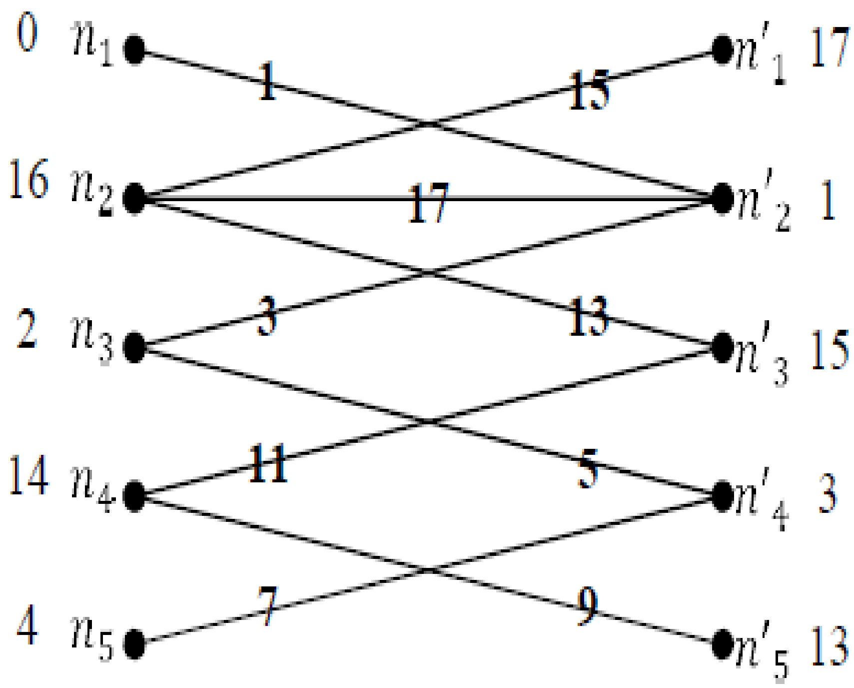

Figure 3.1: Odd Elegant Labeling in P_4

3.0.5 Theorem

Theorem 3.0.2. *The star graph $S_m, m \geq 2$, admits odd elegant labeling.*

Proof. The $2m$ vertices of the star graph are labeled using $0, 1, 2, ..., 4m - 3$. The induced function $\Phi^\star$ assigns to the m edges $e_1, e_2, e_3, ..., e_{m-1}$ labeled as $1, 3, 5, 7, ..., 2m - 1$ respectively and to the $m - 1$ edges $e'_1, e'_2, e'_3, ..., e'_{m-1}$ labeled as $2m + 1, 2m + 3, 2m + 5, ..., 4m - 3$ respectively.

Thus the $2m - 1$ edges are labeled with $1, 3, 5, 7, ..., 4m - 3$.

Hence the graph $S_m, m \geq 2$ admits odd elegant labeling. □

3.0.6 Algorithm-OELSG

$V \leftarrow \left\{v_1, v_2, ..., v_m, v_1^{'}, v_2^{'}, ..., v_m^{'}\right\}$
$E \leftarrow \left\{e_1, e_2, ..., e_m, e_1^{'}, e_2^{'}, ..., e_{m-1}^{'}\right\}$

Fix $v_1 \leftarrow 0$

For $1 \leq k \leq m$
$v_k^{'} \leftarrow 2k - 1$

For $1 \leq k \leq m - 1$
$v_{k+1} \leftarrow 2m + 2k - 2$

3.0.7 Illustration

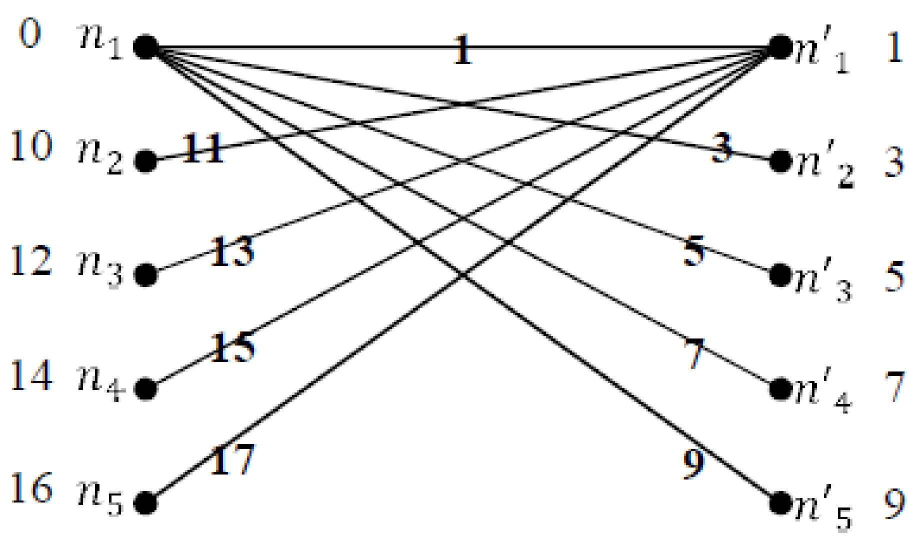

Figure 3.2: Odd Elegant Labeling in S_5

3.0.8 Theorem

Theorem 3.0.3. *The ladder graph $L_m, m \geq 2$, admits odd elegant labeling.*

Proof. The m vertices of the ladder graph namely $v_1, v_3, v_5, v_7, ...v_{2m-1}$ are labeled with $0, 3, 6, 9, ...3m - 3$ respectively and the remaining m vertices namely $v_2, v_4, v_6, v_8, ...v_{2m}$ are

labeled with $1, 4, 7, 10, ...3m - 2$ respectively.
Using the induced function $\Phi^{\star}$ defined by $\Phi^{\star}(uv) = \Phi(u) + \Phi(v)$ the edges are labeled as below.
$\Phi^{\star}(v_{2i-1}v_{2i+1}) = 6i - 3; (1 \le i \le m - 1)$
$\Phi^{\star}(v_{2i}v_{2i+2}) = 6i - 1; (1 \le i \le m - 1)$
$\Phi^{\star}(v_{2i-1}v_{2i}) = 6i - 5; (1 \le i \le m)$

Thus, the $3m - 2$ edges are labeled with $1, 3, 5, 2q - 1$.

Hence, the ladder graph $L_m, m \ge 2$ admits odd edge labeling.

□

3.0.9 Illustration

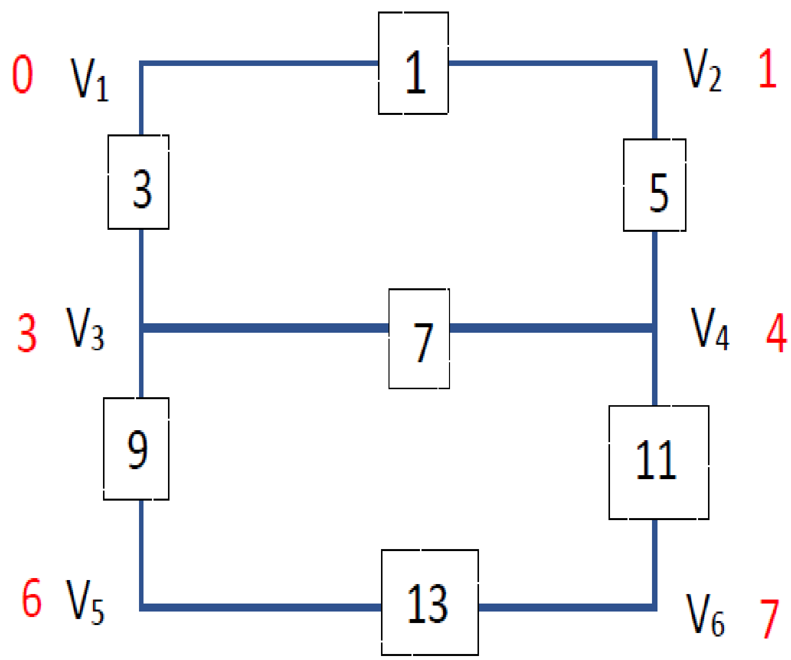

Figure 3.3: Odd Elegant labeling in L_3

3.0.10 Theorem

Theorem 3.0.4. *For $m \geq 2$, the duplicate graph of ladder graph admits odd edge labeling.*

Proof. Using the algorithm, the $4m$ vertices of the duplicate graph of ladder graph $DG(L_m)$ are labeled using $\{0, 1, 2, ...2q-1\}$.

Case(1): When m is odd.
The induced function $\Phi^{\star}$ is defined by $\Phi^{\star}(uv) = \Phi(u) + \Phi(v)$ assigns the labels to the edges as follows.

The $3m-2$ edges namely $e_1, e_2, e_3^{'}, e_4^{'}, e_5^{'}, e_6, e_7, e_8, e_9^{'}, e_{10}^{'}, e_{11}^{'}, e_{12}, e_{13}, e_{14}, ..., e_{3m-9}, e_{3m-8}, e_{3m-7}, e_{3m-6}^{'}, e_{3m-5}^{'}, e_{3m-4}^{'}, e_{3m-3}, e_{3m-2}$ receives the labels $1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, ..., q-15, q-13, q-11, q-9, q-7, q-5, q-3, q-1$ respectively and the remaining $3m-2$ edges namely $e_1^{'}, e_2^{'}, e_3, e_4, e_5, e_6^{'}, e_7^{'}, e_8^{'}, e_9, e_{10}, e_{11}, e_{12}^{'}, e_{13}^{'}, e_{14}^{'}, ..., e_{3m-9}^{'}, e_{3m-8}^{'}, e_{3m-7}^{'}, e_{3m-6}, e_{3m-5}, e_{3m-4}, e_{3m-3}^{'}, e_{3m-2}^{'}$ receives the labels $q+1, q+3, q+5, q+7, q+9, q+11, ..., 2q-9, 2q-7, 2q-5, 2q-3, 2q-1$ respectively.

Hence the $6m-4$ edges are labeled with $1, 3, 5, ..., 2q-1$.

Case(2): When m is even.
The induced function $\Phi^{\star}$ is defined by $\Phi^{\star}(uv) = \Phi(u) + \Phi(v)$ assigns the labels to the edges as follows.

The $3m-2$ edges namely $e_1, e_2, e_3^{'}, e_4^{'}, e_5^{'}, e_6, e_7, e_8, e_9^{'}, e_{10}^{'}, e_{11}^{'}, e_{12}, e_{13}, e_{14}, ..., e_{3m-12}, e_{3m-11}, e_{3m-10}, e_{3m-9}^{'}, e_{3m-8}^{'}, e_{3m-7}^{'}, e_{3m-6}, e_{3m-5}, e_{3m-4}, e_{3m-3}^{'}, e_{3m-2}^{'}$ receives the labels $1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, ..., q-21, q-19, q-17, q-15, q-13, q-11, q-9, q-7, q-5, q-3, q-1$ respectively and the remaining $3m-2$ edges namely $e_1^{'}, e_2^{'}, e_3, e_4, e_5, e_6^{'}, e_7^{'}, e_8^{'}, e_9, e_{10}, e_{11}, e_{12}^{'}, e_{13}^{'}, e_{14}^{'}, ..., e_{3m-12}^{'}, e_{3m-11}^{'}, e_{3m-10}^{'}, e_{3m-9}, e_{3m-8}, e_{3m-7}, e_{3m-6}^{'}, e_{3m-5}^{'}, e_{3m-4}^{'}, e_{3m-3}, e_{3m-2}$ receives the labels $q+1, q+3, q+5, q+7, q+9, q+11, ..., 2q-21, 2q-19, 2q-17, 2q-15, 2q-13, 2q-11, 2q-9, 2q-7, 2q-5, 2q-3, 2q-1$ respectively.

Hence the $6m-4$ edges are labeled with $1, 3, 5, ..., 2q-1$. □

3.0.11 Algorithm-OELDLG

$$V \leftarrow \left\{v_1, v_2, v_3, ..., v_{2m}, v_1^{'}, v_2^{'}, v_3^{'}, ..., v_{2m}^{'}\right\}$$
$$E \leftarrow \left\{e_1, e_2, e_3, ..., e_{3m-2}, e_1^{'}, e_2^{'}, e_3^{'}, ..., e_{3m-2}^{'}\right\}$$

Case(1): When m is odd.

Fix $v_1 \leftarrow 1; v_1^{'} \leftarrow (\frac{q}{2}) + 1$

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For $1 \leq k \leq \frac{m-1}{2}$

$v_k \leftarrow 6k - 1$
$v_{4k+1} \leftarrow 6k + 1$
$v_{4k-2} \leftarrow \frac{q}{2} + (6k - 6)$
$v_{4k-1} \leftarrow \frac{q}{2} + (6k - 4)$

$v^{'}_{4k-2} \leftarrow 6k - 6$
$v^{'}_{4k-1} \leftarrow 6k - 4$
$v^{'}_{4k} \leftarrow \frac{q}{2} + (6k - 1)$
$v^{'}_{4k+1} \leftarrow \frac{q}{2} + (6k + 1)$

For $k = 2m$
$v_k \leftarrow q - 1$
$v^{'}_k \leftarrow \frac{q}{2} - 1$

Case(2): When m is even.

Fix $v_1 \leftarrow 1; v^{'}_1 \leftarrow (\frac{q}{2}) - 1$

For $1 \leq k \leq \frac{m}{2} - 1$

$v_k \leftarrow 6k - 1$
$v_{4k+1} \leftarrow 6k + 1$
$v^{'}_{4k} \leftarrow \frac{q}{2} + (6k - 3)$
$v^{'}_{4k+1} \leftarrow \frac{q}{2} + (6k - 1)$

For $1 \leq k \leq \frac{m}{2}$

$v_{4k-2} \leftarrow \frac{q}{2} + (6k - 4)$
$v_{4k-1} \leftarrow \frac{q}{2} + (6k - 2)$
$v^{'}_{4k-2} \leftarrow (6k - 6)$
$v^{'}_{4k-1} \leftarrow (6k - 4)$

For $k = 2m$
$v_k \leftarrow \dfrac{q}{2} + 1$
$v^{'}_k \leftarrow q - 1$

3.0.12 Illustration

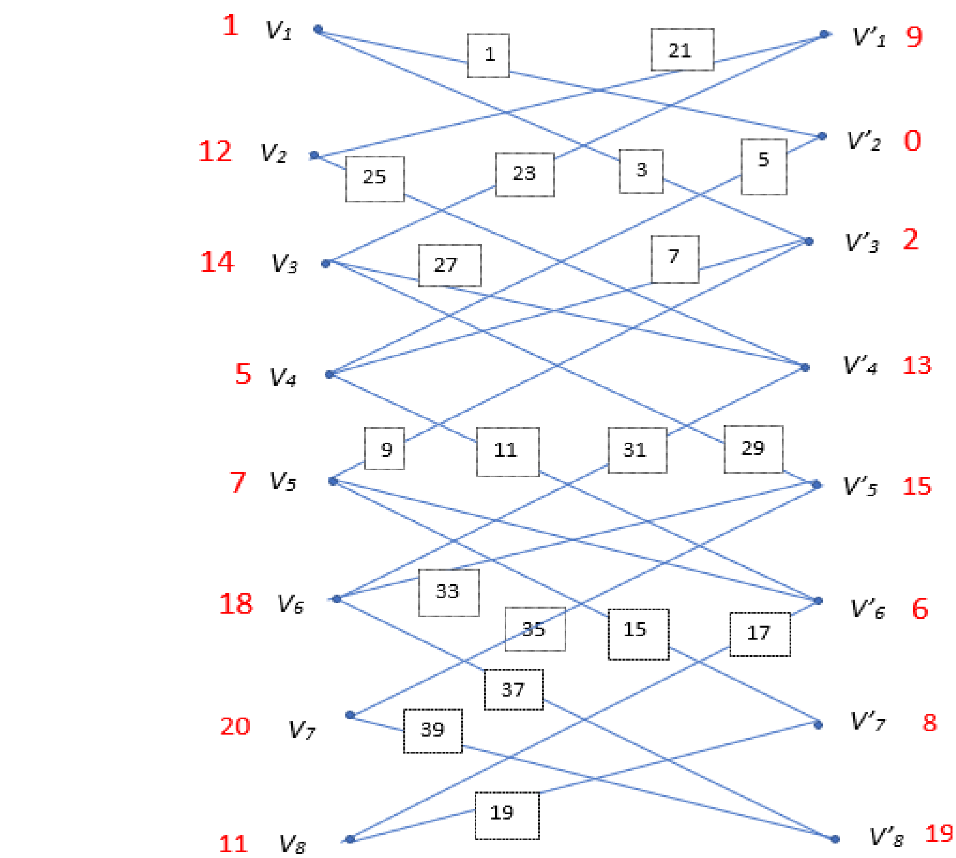

Figure 3.4: Odd Elegant Labeling in DL_4

Chapter 4

4.0.1 Conclusion

We have proved that the path graph, star graph, ladder graph and duplicate graph of ladder graph admits odd edge and odd elegant labeling.

4.0.2 Future Scope

By using this, this can be extended for the examination of existance of odd edge labeling and odd elegant labeling in duplicate graph of cyclic graphs.

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