A NEW WEIGHTED MEASURE OF FUZZY DIRECTED DIVERGENCE

RUCHI HANDA[1*]

[1]Department of Applied Sciences, Khalsa College of Engineering and Technology, Amritsar, India.*

Abstract: *In various fields of science, divergence measures have a very important place. There already exist many standard probabilistic measures of divergence but still there is possibility that some new measures can be constructed so as to provide their applications in various fields. We apply the same criteria for the fuzzy distributions by taking into consideration the importance of fuzzy events. In the present communication, we have derived a new weighted measure and discussed the detailed properties for its authenticity.*

Keywords: **Distance measure, Weighted measure, Fuzzy distribution, Weighted distribution.**

1. INTRODUCTION

Kullback and Leibler [11] measure is expressed as:

$$D(P:Q)=\sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i} \tag{1.1}$$

In the present communication, we extend the concept of distance measure for discrete fuzzy distributions. Yager [15] and Kaufmann [9] have too presented this concept. Kapur [8] expressed many generalized divergence measures dealing with fuzzy distributions and such expressions correspond to existing measures of Harvada and Charvat [4], Renyi [13], Kapur [7], Sharma and Taneja [14] etc. Considering the idea by Zadeh [16]. Other measures are dueee to Bhandari, Pal and Majumde [1], Fan, Ma and Xie [2], Joshi and Kumar [5,6], Markechová, Mosapour and Ebrahimzadeh [12], Kobza [10] etc.

The notion of fuzzy sets provides a convenient point of departure for the construction of a conceptual frame-work. Applying the idea of fuzzy sets, and the concept of weighted information introduced by Guiasu [3], we generated a measure.

2. DEVELOPMENT OF A NEW WEIGHTED MEASURE OF FUZZY DIVERGENCE

Our objective is to introduce a new weighted divergence measure given by:

$$D(A,B;W)=\sum_{i=1}^{n} w_i \left[\begin{array}{l} \left\{ \mu_A(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)} - 1-\mu_A(x_i)\log\frac{1-\mu_A(x_i)}{1-\mu_B(x_i)} \right\} \\ \qquad + \frac{\left(\mu_A(x_i)-\mu_B(x_i)\right)^2}{\left(\mu_A(x_i)+\mu_B(x_i)\right)} + \frac{\left(\mu_B(x_i)-\mu_A(x_i)\right)^2}{\left(2-\mu_A(x_i)-\mu_B(x_i)\right)} \end{array} \right] \quad (2.1)$$

$$+\frac{1}{2}\sum_{i=1}^{n} w_i \left[\left(\sqrt{\mu_A(x_i)}-\sqrt{\mu_B(x_i)}\right)^2 + \left(\sqrt{1-\mu_A(x_i)}-\sqrt{1-\mu_B(x_i)}\right)^2 \right]$$

Next, we study the properties of the proposed measure (2.1).

Convexity: We have

$$\frac{d^2 D(A,B;W)}{d\mu_A^{\,2}(x_i)}$$

$$= w_i\left\{\frac{1}{\mu_A(x_i)}+\frac{1}{1-\mu_A(x_i)}\right\}+\frac{8w_i\mu_B^{\,2}(x_i)}{\left(\mu_A(x_i)+\mu_B(x_i)\right)^3}+\frac{2w_i\left(4+4\mu_B^{\,2}(x_i)-8\mu_B(x_i)\right)}{\left(2-\mu_A(x_i)-\mu_B(x_i)\right)^3}$$

$$+\frac{1}{4}w_i\left[\frac{\sqrt{\mu_B(x_i)}}{\left(\mu_A(x_i)\right)^{\frac{3}{2}}}+\frac{\sqrt{1-\mu_B(x_i)}}{\left(1-\mu_A(x_i)\right)^{\frac{3}{2}}}\right]>0$$

which proves the convexity.

Again, we have

$$\frac{d^2 D(A,B;W)}{d\mu_B^{\,2}(x_i)} = w_i\left\{-\frac{\mu_A(x_i)}{\mu_B^2(x_i)}+\frac{1-\mu_A(x_i)}{\left(1-\mu_B(x_i)\right)^2}\right\}+\frac{8w_i\mu_A^{\,2}(x_i)}{\left(\mu_A(x_i)+\mu_B(x_i)\right)^3}$$

$$+\frac{2w_i\left(4+4\mu_A^{\,2}(x_i)-8\mu_A(x_i)\right)}{\left(2-\mu_A(x_i)-\mu_B(x_i)\right)^3}+\frac{1}{4}w_i\left[\frac{\sqrt{\mu_A(x_i)}}{\left(\mu_B(x_i)\right)^{\frac{3}{2}}}+\frac{\sqrt{1-\mu_A(x_i)}}{\left(1-\mu_B(x_i)\right)^{\frac{3}{2}}}\right]>0$$

which proves the convexity with respect to $\mu_B(x_i)$.

Also,

1. $D(A,B;W)\geq 0$
2. $D(A,B;W)=0$ iff A=B.
3. $D(A,B;W)$ does not change when $\mu_A(x_i)$ is replaced by $1-\mu_A(x_i)$ and $\mu_B(x_i)$ is replaced by $1-\mu_B(x_i)$.

4. $D(A,B;W)$ is a convex function of $\mu_A(x_i)$ and $\mu_B(x_i)$.

So, the divergence measure (2.1) is a valid measure of directed divergence.

Further, there is graphical presentation of the divergence measure (2.1) with the following table- 2.1.

Table-2.1. Graphical presentation of D (A, B; W)

$\mu_A(x_i)$	w_i	$D(A,B;W)$
0.1	1.1	0.9400048337
0.2	1.15	0.508143205
0.3	1.2	0.224051879
0.4	1.25	0.056751726
0.5	1.28	0
0.6	1.25	0.056751726
0.7	1.2	0.224051879
0.8	1.15	0.508143205
0.9	1.1	0.940048337

From the above table, we have obtained the following Fig.-2.1 which shows that the divergence measure introduced in equation (2.1) is a convex function.

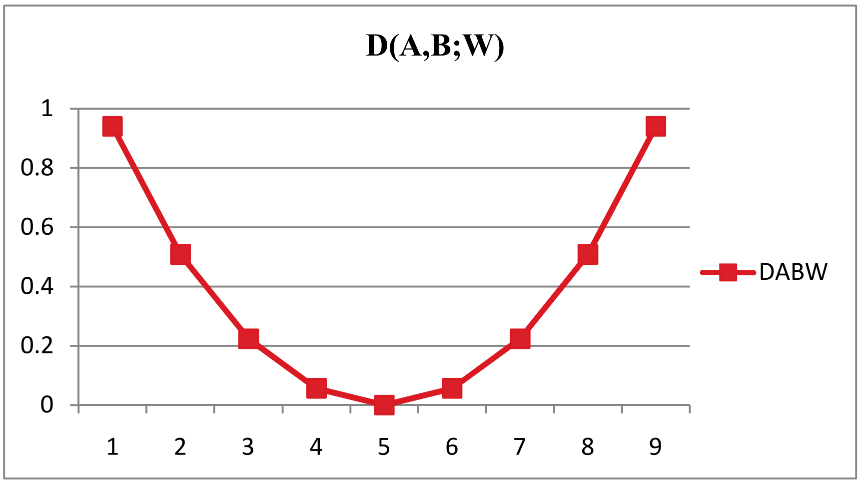

Fig.-2.1. Representation of D (A, B; W)

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REFERENCES

[1] *Bhandari, D., Pal, N.R. and Majumder, D.D, "Fuzzy divergence, probability measure of fuzzy events and image thresholding", Pattern Recognition Letters., 1,* ***(1992)****, pp. 857-867.*

[2] *Fan, J.L., Ma, Y.L. and Xie, W.X, "On some properties of distance measure", Fuzzy Sets and Systems., 117,* ***(2001)****, pp. 355-361.*

[3] *Guiasu, S, "Weighted entropy", Reports on Mathematical Physics., 2,* ***(1971)****, pp. 165-171.*

[4] *Havrada, J.H. and Charvat, F, "Quantification methods of classification process: Concept of structural α-entropy", Kybernetika., 3,* ***(1967)****, pp. 30-35.*

[5] *Joshi, R. and Kumar, S, "An exponential Jensen fuzzy divergence measure with applications in multiple attribute decision-making",* Math. Probl. Eng. *Art. ID 4342098,* ***(2018)****.*

[6] *Joshi, R. and Kumar, S, "An (R', S')-norm fuzzy relative information measure and its applications in strategic decision-making",* Comput. Appl. Math. *37 (4),* ***(2018)****, pp. 4518–4543.*

[7] *Kapur, J.N, "Measures of Information and Their Application", Wiley Eastern, New York,* ***(1995)****.*

[8] *Kapur, J.N, "Measures of Fuzzy Information", Mathematical Sciences Trust Society, New Delhi,* ***(1997)****.*

[9] *Kaufmann, A, "Introduction to Theory of Fuzzy Subsets," Academic Press, New York,* ***(1975)****.*

[10] *Kobza, V, "Divergence measure between fuzzy sets using cardinality",* Kybernetika*, pp. 418–436.*

[11] *Kullback, S. and Leibler, R.A, "On information and sufficiency", Ann. Maths. Stat. 22,* ***(1951)****, pp. 79-86.*

[12] *Markechová, D., Mosapour, B. and Ebrahimzadeh, A, "R-norm entropy and R-norm divergence in fuzzy probability spaces",* Entropy *20 (4), Paper No. 272,* ***(2018)****, 18 pp.*

[13] *Renyi, A, "On measures of entropy and information", Proceedings 4th Berkeley Symposium on Mathematical Statistics and Probability, 1,* ***(1961)****, pp. 547-561.*

[14] *Sharma, B.D. and Taneja, I.J, "Entropies of type (α,β) and other generalized measures of information theory", Metric, 22,* ***(1975)****, pp. 202-215.*

[15] *Yager, R, "On measures of fuzziness and negation, Part-I: membership in the unit interval", International Journal of General Systems, 5,* ***(1979)****, pp. 221-229.*

[16] *Zadeh, L.A, "Fuzzy sets", Inform. and Control 8,* ***(1965)****, pp. 338-353.*