MODELING THE VELOCITY OF A VERTICALLY LAUNCHED SINGLE-STAGE ROCKET WITH TIME-VARYING MASS

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Abstract: *This paper presents a mathematical model for the velocity v(t) of a small single-stage rocket launched vertically. The rocket's mass m(t) decreases linearly over time due to fuel consumption. The model accounts for three primary forces: the constant upward thrust R from the propulsion system, the air resistance proportional to the rocket's instantaneous velocity kv(t), and the downward gravitational force m(t)g. By applying Newton's second law of motion, we derive a first-order linear differential equation for v(t) with time-varying coefficients. The resulting equation:*

$$\frac{dv(t)}{dt}+\frac{k}{m_0-\alpha t}v(t)=\frac{R}{m_0-\alpha t}-g$$

is solved using standard techniques for differential equations, providing a comprehensive model for the rocket's velocity over time. This model is crucial for understanding the dynamics of rocket flight and optimizing propulsion and fuel efficiency in aerospace engineering.

Keywords: *Rocket dynamics, Velocity model, Time-varying mass, Air resistance, Thrust, Gravitational force, Differential equation, Aerospace engineering, Propulsion system, Fuel consumption.*

1. INTRODUCTION

1.1 Newton's second law of motion indicates that when the net force acting on a body is not zero, then the net force is proportional to its acceleration a or, more precisely $F = ma$, where m is the mass of the body.

1.2 Newton's Second law and Rocket Motion: When the mass m of a body is changing with time, Newton's second law of motion becomes

$$F=\frac{d}{dt}(mv)$$

Where F is the net force acting on the body and mv is its momentum.

1.3 consider the forces acting on the rocket:

1. Thrust R generated by the propulsion system(assumed to be constant)
2. Air resistance, which is proportional to the instantaneous velocity (kv(t), where k is a positive constant).

3. Gravity (mg(t), where g is the acceleration due to gravity and m(t) is the mass of the rocket at time t).

2. STATEMENT OF THE PROBLEM

2.1 A small single-stage rocket is launched vertically as shown in the figure given below. Once launched, the rocket consumes its fuel, and so its total mass m(t) varies with time $t > 0$. If it is assumed that the positive direction is up-ward, air resistance is proportional to the instantaneous velocity v of the rocket, and R is the upward thrust or force generated by the propulsion system, then constructing a mathematical model for the velocity v(t) of the rocket.

Figure 1

2.2 MATHEMATICAL MODEL FOR THE VLOCITY V(T) OF THE ROCKET:

Using Newton's second law of motion, the net force on the rocket equals the mass times the acceleration:

$$F_{net} = m(t)\,\frac{dv(t)}{dt}$$

The net force is the sum of the thrust, the air resistance(which acts in the opposite direction of the motion), and the gravitational force(which acts downward):

$$R - kv(t) - m(t)g - m(t)\frac{dv(t)}{dt}$$

$$\Rightarrow m(t)\frac{dv(t)}{dt} = R - kv(t) - m(t)g \ldots\ldots\ldots\ldots\text{(i)}$$

To solve this equation, we need to account for the time-varying mass m(t).

Assume that the mass decreases linearly with time due to fuel consumption:

$m(t) = m_0 - at$

Where m_0 is the initial mass of the rocket

α is the rate at which mass is lost per unit time.

Substituting m(t) into the differential equation(i), we get

$$(m_0 - \alpha t)\frac{dv(t)}{dt} = R - kv(t) - (m_0 - \alpha t)g$$

$$\frac{dv(t)}{dt} = \frac{R - kv(t) - (m_0 - \alpha t)g}{m_0 - \alpha t}$$

This is a first-order linear differential equation in v(t). To solve it, we can use an integrating factor or other appropriate methods for first-order linear Ordinary Differential Equations.

For simplicity, the equation in standard form:

$$\frac{dv(t)}{dt} + \frac{k}{m_0 - \alpha t}v(t) = \frac{R}{m_0 - \alpha t} - g$$

This differential equation can now be solved using standard techniques for linear differential equations with variable coefficients. The solution will provide the velocity v(t) as a function of time

3.CONCLUSION

In this study, we developed a mathematical model to describe the velocity v(t) of a vertically launched single-stage rocket with time-varying mass. By considering the forces of constant thrust R, proportional air resistance kv(t), and gravitational pull m(t)g, we formulated a first-order linear differential equation. This equation accurately represents the dynamics of the rocket's motion as it ascends, factoring in the gradual depletion of fuel and its consequent effect on mass reduction. Solutions to this model provide insights into optimizing rocket performance and trajectory prediction, crucial for advancements in aerospace engineering and space exploration missions. Future research may explore further refinements to accommodate varying atmospheric conditions and more complex propulsion systems for enhanced accuracy and efficiency in rocket design.

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