MDH Laminer Source Flow Between Porous Disks

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Abstract:

*Navier-Stokes equations have been solved for the problem of conducting laminar source flow between two infinite non-conducting disks. The solution obtained embodies the solution for the source free flow between porous disks as well as that for source flow between non-porous disks. *. In magnetohydrodynamics, the source flow has not been given much attention.The solution for the MHD laminar source flow is obtained by perturbating the creeping flow solution that is valid for small value reduced Reynolds number R_e^*. The asymptotic solutions are also obtained for creeping flow and other velocity perturbation. Expressions for velocity, pressure and shear stress are obtained.*

Key Words: Velocity, stress, Renolds number, Pressure,NS-Equation

1. INTRODUCTION

The steady laminar source flow of an incompressible conducting viscous fluid between two porous disks is considered in the presence of a transverse magnetic field. In hydrodynamics the laminar source flow between two parallel stationary non porous disks has been examined by Peube [1] and Savage [2] and Sourieau [3] and their theoretical result agree with experimental results of moller [4]. Terril and Cornish [5] have considered the radial flow of a viscous incompressible fluid between stationary porous disks. The steady flow between porous disks numerically has been discussed by Rasmussen [6]. Wang and Watson [7] have investigated the radial flow between rotating disks with injection on the porous disk and made a comparison of the previously known analytical result with numerical result. Source flow between two parallel non-porous disks rotating at the same velocity has been investigated by Breitner and Polhausen [8] and by Peube and kreith [9]. The same problem with disks rotating at different speeds has been studied by Kreith and Viviand [10]. Elkouh [11] has investigated the laminar source flow between parallel stationary porous disks where he obtains a solution which is valid for small values of wall Reynolds number R_W and for small value of reduced Reynolds number R_e^*.

The result of the present investigation may find application in gaseous diffusion, boundary cooling and lubrication of porous MHD bearing etc.

2. FORMULATION OF THE PROBLEM

We consider cylindrical polar coordinates $(\overline{r},\overline{\theta},\overline{z})$ and A uniform incompressible fluid having density ρ, kinematic viscosity ν, electrical conductivity, σ the axially symmetric steady flow between two infinite stationary porous disks. Which is lie in the plane $\overline{z}=-a$ and $\overline{z}=+a$ (fig. 1) $\overline{u},\overline{v}$ are velocity components in the directions $\overline{r}$ and $\overline{z}$. So that $\overline{u}=\overline{u}(\overline{r}\,\overline{z})$ and $\overline{v}=\overline{v}(\overline{r}\,\overline{z})$. A source of strength Q is placed at the centre of the channel formed by the disks

The governing equation of motion of steady, axisymmetric flow, for small magnetic Reynolds number based on the radius L are

$$\overline{u}\frac{\partial\overline{u}}{\partial\overline{r}}+\overline{v}\frac{\partial\overline{u}}{\partial\overline{z}}=-\frac{1}{\rho}\frac{\partial\overline{p}}{\partial\overline{r}}+\nu\left(\frac{\partial^2\overline{u}}{d\overline{r}^2}+\frac{1}{\overline{r}}\frac{\partial\overline{u}}{\partial\overline{r}}-\frac{\overline{u}}{\overline{r}^2}+\frac{\partial^2\overline{u}}{\partial\overline{z}^2}\right)-\sigma\frac{B_0^2\overline{u}}{\rho} \quad \text{…(2.2A)}$$

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{r}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{z}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{z}} + \nu\left(\frac{\partial^2 \bar{v}}{d\bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2}\right) \quad \ldots(2.2B)$$

$$\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{u}}{\bar{r}} + \frac{\partial \bar{v}}{\partial \bar{z}} = 0 \quad \ldots(2.2C)$$

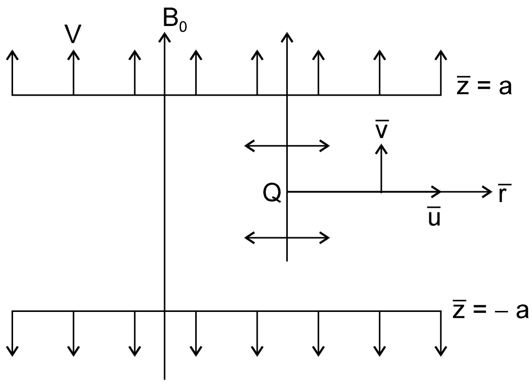

Fig. 1 : Flow Between Porous Disks

The fluid is injected or extracted with uniform velocity V at the disks.

The boundary conditions are

$$\bar{u}(r, \pm a) = 0$$

$$\bar{v}(r, \pm a) = \mp \nu \quad \text{and}$$

$$\int_{-a}^{a} 2\pi \bar{r}\,\bar{u}\,d\bar{z} - 2\pi \bar{r}^2 V = Q \quad \ldots(2.2D)$$

We introduce the following dimensionless quantities

$$r = \frac{\bar{r}}{a} \qquad z = \frac{\bar{z}}{a}$$

$$u = \frac{\bar{u}a}{\nu} \qquad v = \frac{\bar{v}a}{\nu}$$

$$p = \frac{\bar{p}a^2}{\rho\nu^2} \quad \ldots(2.2E)$$

with the help of (2.2E) the governing equation of motion of steady reduce to

$$u\frac{\partial u}{\partial r} + V\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial r} + \left(\frac{\partial^2 u}{dr^2} + \frac{1}{\bar{r}}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) - M^2 u \quad \ldots(2.2F)$$

$$u\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial z} + \left(\frac{\partial^2 v}{dr^2} + \frac{1}{r^2}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2}\right) \quad \ldots(2.2G)$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial v}{\partial z} = 0 \quad \ldots(2.2H)$$

where $M^2 = \frac{\sigma B_0^2 h^2}{\rho\nu}$ is Hartmann number square

equation (2.2D) and (2.2E) give the modified boundary conditions in the following form

$$\left.\begin{aligned} & u(r,\pm 1)=0 \\ & v(r,\pm 1)=\mp R_W=\mp\frac{M^2}{N} \\ & \int_{-1}^{1} u\,dz=\frac{M^2 r}{N}+\frac{2R_e}{r} \end{aligned}\right\} \quad \ldots(2.2I)$$

where $N=\dfrac{\sigma B_0^2 h}{\rho V}$ is interaction parameter,

R_e is source flow Reynolds number

and R_W = is the well Reynolds number, which is taken positive for injection and negative for suction.

Now we introduce a dimensionless stream function in the form

$$u=\frac{1}{r}\frac{\partial\psi}{\partial z} \quad \ldots(2.2J)$$

$$\nu=-\frac{1}{r}\frac{\partial\psi}{\partial r} \quad \ldots(2.2K)$$

Satisfying the equation of continuity.

We assume the following expansion for ψ and P which are vaid for small values of $R_e^*\left(=\dfrac{R_e}{r^2}\right)$, that is, at large distance from the source

$$\psi=\frac{1}{2}r^2\frac{M^2}{N}f_{-1}(z)+R_e\left[f_0(z)+\frac{R_e}{r^2}f_1(z)+\cdots\right] \quad \ldots(2.2L)$$

and $$p=\frac{1}{4}r^2\frac{M^2}{N}h_{-1}(z)+h(z)+R_e\left[h_0(z)\ln r+\frac{R_e}{r^2}h_1(z)+\cdots\right] \quad \ldots(2.2M)$$

equation (2.2J), (2.2K) and (2.2L) give the expansions for velocity components u and v

$$u=\frac{1}{2}r\frac{M^2}{N}f'_{-1}+\frac{R_e}{r}\left[f'_0+\frac{R_e}{r^2}f'_1+\cdots\right] \quad \ldots(2.2N)$$

and $$\nu=-\frac{M^2}{N}f_{-1}+R_e\left[2\left(\frac{R_e}{r^2}\right)^2 f_1+\cdots\right] \quad \ldots(2.2O)$$

where the primes denote differentiation with respect to z.

Equations (2.2I), (2.2J), (2.2K) and (2.2L) give the modified boundary conditions for the functions f_n and their derivatives are –

$$f'_n(\pm 1)=0 \qquad n=-1,0,1,2,\cdots$$

$$f_n(\pm 1)=0 \qquad n=1,2,\cdots$$

$$f_{-1}(\pm 1)=\pm 1$$

and $$F_0(1)-F_0(-1)=2 \quad \ldots.(2.2P)$$

we choose

$$f_0(-1)=-1,\ \text{so that}\ f_0(1)=1 \quad \ldots(2.2Q)$$

To obtain the solution for the above set of equations, we substituting the expansions of p, u and v in equations (2.2F) and (2.2G) and equating the coefficients of like powers of R_e. This gives us an infinite set of system of simultaneous ordinary differential equation of which the first system is non-linear. The first three system considered here are :

System – I :

$$f'''_{-1} + \frac{M^2}{N}\left[f_{-1}f''_{-1} - \frac{1}{2}f'^2_{-1}\right] = h_{-1} + M^2 f'_{-1}$$

h_{-1} = Constant …(2.2R)

and $$h = -\frac{M^2}{N}\left[f'_{-1} + \frac{1}{2}\frac{M^2}{N}f^2_{-1}\right] + \text{Constant} \quad \ldots(2.2S)$$

where the constant is determined from a known pressure at a point in the flow.

System – II :

$$f'''_0 + \frac{M^2}{N}f_{-1}f''_0 = h_0 + M^2 f'_0$$

h_0 = constant …(2.2T)

System – III :

$$f'''_1 + \frac{M^2}{N}\left(f'_{-1}f'_1 + f_{-1}f''_1 - f_1 f''_{-1}\right) = -2h_1 - f'^2_0 + M^2 f'_1$$

h_1 = constant …(2.2U)

3. SOLUTION OF THE PROBLEM

Asymptotic solution for $R_W << M^2$

The sets of differential equation in the above system have been solved by a perturbation scheme, expanding the different unknown function f_n and h_n in the powers of $\frac{1}{N}$ as follows :

$$f_n = \sum_{\alpha=0}^{\infty}\frac{1}{N^{\alpha}}f_{n,\alpha}$$

and $$h_n = \sum_{\alpha=0}^{\infty}\frac{1}{N^{\alpha}}f_{n,\alpha} \quad \ldots(2.3A)$$

where $f_{n,\alpha}$ and $h_{n,\alpha}$ are independent of N.

The boundary conditions to be satisfied by $f_{n,\alpha}$ are

$$f'_{n,\alpha}(\pm 1) = 0 \quad \text{for } n = -1, 0, 1, 2 \text{ and all } \alpha$$

$$f_{n,\alpha}(\pm 1) = 0 \quad \text{for } n = 1, 2 \text{ and all } \alpha$$

$$f_{n,0}(\pm 1) = \pm 1 \quad \text{for } n = -1, 0$$

$$f_{n,\alpha}(\pm 1) = 0 \quad \text{for } n = -1, 0 \text{ and } \alpha \geq 1 \quad \ldots(2.3B)$$

Solution for system I :

Since from equation (2.3A) we have

$$f_{-1} = f_{-1,0} + \frac{1}{N}f_{-1,1} + \frac{1}{N^2}f_{-1,2} + \frac{1}{N^3} + f_{-1,3} + \cdots \quad \ldots(2.3C)$$

and $$h_{-1} = h_{-1,0} + \frac{1}{N}h_{-1,1} + \frac{1}{N^2}h_{-1,2} + \frac{1}{N^3} + h_{-1,3} + \cdots \quad \ldots(2.3D)$$

Substituting the expansions of f_{-1} and h_{-1} from (2.3C) and (2.3D) in the equation (2.2R) and equating the coefficient of like power of 1/N. The gives us the following set of linear ordinary differential equations

$$f'''_{-1,0} - M^2 f'_{-1,0} \approx h_{-1,0} \quad \ldots(2.3E)$$

$$f'''_{-1,1} - M^2 f'_{-1,1} = h_{-1,1} + M^2\left(\frac{1}{2}f'^2_{-1,0} - f_{-1,0}f''_{-1,0}\right) \quad \ldots(2.3F)$$

The analysis is confined to first order, since the second order perturbation is algebraically complicated and also the effect of second order terms compared to first order terms are negligible.

Solving (2.3E), we get

$$f_{-1,0}(z) = C_1 + C_2 \cos h\, Mz + C_3 \sin h\, Mz - \frac{h_{-1,0} z}{M^2} \quad \ldots(2.3G)$$

Constant are determined by using the boundary conditions (2.3B) that are

$$f_{-1,0}(1) = 1 \quad ; f_{-1,0}(-1) = -1$$

$$f'_{-1,0}(1) = 0 \quad ; f'_{-1,0}(-1) = 0$$

Solution (2.3G) satisfying these conditions is

$$f_{-1,0} = \frac{A}{M}\left(Mz \cos h\, M - \sin h\, Mz\right) \quad \ldots(2.3H)$$

$$f'_{-1,0} = A\left(\cos h\, M - \cos h\, Mz\right) \quad \ldots(2.3I)$$

where $A = \dfrac{M}{M \cos h\, M - \sin h\, M}$

and $h_{-1,0} = \dfrac{M^3}{\tan h\, M - M}$

Again solving (2.3F), we get

$$f_{-1,1}(z) = C_4 + C_5 \cosh Mz + C_6 \sinh Mz + a_1$$

$$\left[a_2 z + a_3 z \cosh Mz - \frac{1}{2}\sinh 2Mz - 9Mz + a_4 z^2 \sinh Mz\right] - \frac{h_{-1,1} z}{M^2} \quad \ldots(2.3J)$$

Constant are determined by using boundary conditions (2.3B) that are

$$f_{-1,1}(1) = 0 \qquad f_{-1,1}(-1) = 0$$

$$f'_{-1,1}(1) = 0 \qquad f'_{-1,1}(-1) = 0$$

Solution (2.3J) satisfying these conditions is

$$f_{-1,1} = D \sin h\, Mz + a_1\left[a_2 z + a_3 z \cosh Mz - \right.$$

$$\left.\frac{1}{2}\sinh 2Mz - 9Mz + a_4 z^2 \sinh Mz\right] - \frac{h_{-1,1} z}{M^2} \quad \ldots(2.3K)$$

$$f'_{-1,1} = DM \cos h\, Mz + a_1\left[a_2 + a_3\left(Mz \sin h\, Mz + \cos h\, Mz\right)\right.$$

$$\left. - M \cos h\, 2Mz - 9M + a_4 (2z \sin h\, Mz - z^2 M \cos h\, Mz)\right] - \frac{h_{-1,1}}{M^2} \quad \ldots(2.3L)$$

Where

$$D = \frac{a_1}{K}\left[a_3 M \sin h\, M - \frac{1}{2}\left(2M \cos h\, 2M - \sin h\, 2M\right)\right.$$

$$\left. + a_4\left(M \cos h\, M + \sin h\, M\right)\right]$$

$$h_{-1,1} = DM^2 \sin h\, M + a_1$$

$$\left[a_2 + a_3 \cos h\, M - \frac{1}{2}\sin h\, 2M - 9M + a_4 \sin h\, M\right]$$

$$a_1 = \frac{A^2}{12M}$$

$$a_2 = -6M \cos h^2 M$$

$$a_3 = -15M \cos h\, M$$

$$a_4 = 3M^2 \cosh M$$
$$K = \sinh M - M\cosh M$$

and $$A = \frac{M}{M\cosh M - \sinh M}$$

Solution for System II :

Since from equation (2.3A), we have

$$f_{-1} = f_{-1,0} + \frac{1}{N} f_{-1,1} + \frac{1}{N^2} f_{-1,2} + \cdots \quad \text{...(2.3M)}$$

$$f_0 = f_{0,0} + \frac{1}{N} f_{0,1} + \frac{1}{N^2} f_{0,2} + \cdots \quad \text{...(2.3N)}$$

and $$h_0 = h_{0,0} + \frac{1}{N} h_{0,1} + \frac{1}{N^2} h_{0,2} + \cdots \quad \text{...(2.3O)}$$

Substituting the expansions of f_{-1}, f_0 and h_0 from (2.3M), (2.3N) and (2.3O) in the equation (2.2T) and equating the coefficient of like power of 1/N. The gives us the following set of linear ordinary differential equations

$$f'''_{0,0} - M^2 f'_{0,0} = h_{0,0} \quad \text{...(2.3P)}$$

$$f'''_{0,1} - M^2 f'_{0,1} = h_{0,1} - M^2 \left(f_{-1,0} f''_{0,0} \right) \quad \text{...(2.3Q)}$$

Solving (2.3P), we get

$$f_{0,0}(z) = C_7 + C_8 \cosh Mz + C_9 \sinh Mz - \frac{h_{0,0} z}{M^2} \quad \text{...(2.3R)}$$

Constant are determined by using boundary conditions (2.3B) that are

$$f_{0,0}(1) = 1 \qquad f_{0,0}(-1) = -1$$
$$f'_{0,0}(1) = 0 \qquad f'_{0,0}(-1) = 0$$

The solution (2.3R) of equation (2.3P) satisfying these boundary conditions is

$$f_{0,0} = \frac{A}{M}\left(Mz\cosh M - \sinh Mz \right) \quad \text{...(2.3S)}$$

$$f'_{0,0} = A(\cosh M - \cosh Mz) \quad \text{...(2.3T)}$$

where $$A = \frac{M}{M\cosh M - \sinh M}$$

and $$h_{0,0} = \frac{M^3}{\tanh M - M}$$

Again solving equation (2.3Q), we get

$$f_{0,1}(z) = C_{10} + C_{11}\cosh Mz + C_{12} \sinh Mz - \frac{h_{0,1} z}{M^2} +$$
$$b_1\left[b_2 z^2 \sinh Mz + b_3 z \cosh Mz - \sinh 2Mz - 6Mz \right] \quad \text{...(2.3U)}$$

Constants are determined by using boundary conditions (2.3B), that are

$$f'_{0,1}(1) = 0 \qquad f'_{0,1}(-1) = 0$$
$$f_{0,1}(1) = 0 \qquad f_{0,1}(-1) = 0$$

The solution (2.3U) of equation (2.3Q) satisfying these boundary conditions is

$$f_{0,1}(z) = E\sinh Mz - \frac{h_{0,1} z}{M^2} + b_1$$
$$\left[b_2 z^2 \sinh Mz + b_3 z\cosh Mz - \sinh 2Mz - 6Mz \right] \quad \text{...(2.3V)}$$

$$f'_{0,1}(z) = EM\cos h\,Mz - \frac{h_{0,1}}{M^2} + b_1\left[b_2\left(2z\sin h\,Mz + z^2 M\cos h\,Mz\right)\right.$$
$$\left. + b_3\left(zM\sinh Mz + \cosh Mz\right) - 2M\cosh 2M - 6M\right] \quad \ldots(2.3W)$$

where

$$E = \frac{b_1}{K}\left[b_2\left(\sinh M + M\cosh M\right) + b_3 M\sinh M - 2M\cosh 2M + \sin h\,2M\right]$$

$$h_{0,1} = EM^2\sinh M + b_1 M^2\left[b_2\sin hM + b_3\cos h\,M - \sin h\,2M - 6M\right]$$

$$b_1 = \frac{A^2}{12M}$$
$$b_2 = 3M^2\cosh M$$
$$b_3 = -9M\cosh M$$
$$K = \sinh M - M\cosh M$$

and $$A = \frac{M}{M\cosh M - \sinh M}$$

Solution for system III :

Since from equation (2.3A), we have

$$f_{-1} = f_{-1,0} + \frac{1}{N}f_{-1,1} + \frac{1}{N^2}f_{-1,2} + \cdots \quad \ldots(2.3X)$$

$$f_0 = f_{0,0} + \frac{1}{N}f_{0,1} + \frac{1}{N^2}f_{0,2} + \cdots \quad \ldots(2.3Y)$$

$$f_1 = f_{1,0} + \frac{1}{N}f_{1,1} + \frac{1}{N^2}f_{1,2} + \cdots \quad \ldots(2.3Z)$$

and

$$h_1 = h_{1,0} + \frac{1}{N}h_{1,1} + \frac{1}{N^2}f_{1,2} + \cdots \quad \ldots(2.3AA)$$

Substituting the expansions of f_{-1}, f_0, f_1 and h_1 from (2.3X), (2.3Y), (2.3Z) and (2.3AA) in equation (2.2U) and equation the coefficient of like power of 1/N. This gives us the following linear ordinary differential equations

$$f'''_{1,0} - M^2 f'_{1,0} = -2h_{1,0} - f'^2_{0,0} \quad \ldots(2.3BB)$$

Solving (2.3BB), we get

$$f_{1,0} = C_{13} + C_{14}\cosh Mz + C_{15}\sinh Mz + \frac{2h_{1,0}z}{M^2}$$
$$+ e_1\left[e_2 z\cosh Mz + e_3 z + 6Mz - \sinh 2Mz\right] \quad \ldots(2.3CC)$$

Constant are determined by using boundary conditions (2.3B) that are

$$f_{0,1}(1) = 0 \qquad f_{0,1}(-1) = 0$$

$$f'_{0,1}(1) = 0 \qquad f'_{0,1}(-1) = 0$$

The solution (2.3CC) of equation (2.3BB) satisfying these boundary conditions is

$$f_{1,0} = F\sin h\,Mz + \frac{2h_{1,0}z}{M^2}$$
$$+ e_1\left[e_2 z\cosh Mz + e_3 z + 6Mz - \sinh 2Mz\right] \quad \ldots(2.3DD)$$

$$f'_{1,0} = FM\cosh Mz + \frac{2h_{1,0}}{M^2}$$
$$+ e_1\left[e_2\left(zM\sinh Mz + \cos Mz\right) + e_3 + 6M - 2M\cosh 2Mz\right] \quad \ldots(2.3EE)$$

where $F = \frac{e_1}{K}\left[e_2 M \sinh M - 2M \cosh 2M + \sinh 2M\right]$

$$\frac{2h_{1,0}}{M^2} = -F \sinh M - e_1\left[e_2 \cosh M + e_3 + 6M - \sinh 2M\right]$$

$$e_1 = \frac{A^2}{12M^3}$$

$$e_2 = 12M \cosh M$$

$$e_3 = 12M \cosh^2 M$$

$$K = \sinh M - M \cosh M$$

and $A = \frac{M}{M \cosh M - \sinh M}$

Using equations (2.3H) and (2.3K) in equation (2.3A)

$$f_{-1} = \frac{A}{M}\left(Mz \cosh M - \sinh Mz\right) + \frac{1}{N}\Big[D \sinh Mz + a_1 \left(a_2 z + a_3 z \cosh Mz - \frac{1}{2}\sinh 2Mz - 9Mz + a_4 z^2 \sinh Mz\right) - \frac{h_{-1,1} z}{M^2}\Big] + \cdots \quad \ldots(2.3FF)$$

Using (2.3S) and (2.3V) in equation (2.3A)

$$f_0 = \frac{A}{M}\left(Mz \cosh M - \sinh Mz\right) + \frac{1}{N}\Big[E \sinh Mz - \frac{h_{0,1} z}{M^2} + b_1\left(b_2 z^2 \sinh Mz + b_3 z \cosh Mz - \sinh 2Mz - 6Mz\right)\Big] + \cdots \quad \ldots(2.3GG)$$

Using (2.3DD) in equation (2.3A)

$$f_1 = F \sinh Mz + \frac{2h_{1,0} z}{M^2} + e_1\left[e_2 z \cosh Mz + e_3 z + 6Mz - \sinh 2Mz\right] + \cdots \quad \ldots(2.3HH)$$

It is observed from the above solution (2.3FF), (2.3GG), (2.3HH) that they are independent of R_e(source Reynolds number). This is because R_e appears neither in the differential equation nor in the boundary conditions f_0, f_{-1}, and f_1 represents the solutions for laminar source flow between porous disks.

4. DISCUSSION OF RESULTS

Velocity Distribution :

The component of radial velocity, in terms of the average radial velocity, is given by

$$u^* = \frac{u}{\langle u \rangle} = \frac{\frac{1}{2}\frac{M^2}{N}\left(f'_{-1}\right) + \frac{R_e}{r^2}\left[f'_0 + \frac{R_e}{r^2} f'_1 + \cdots\right]}{\left(\frac{1}{2}\frac{M^2}{N} r + \frac{R_e}{r^2}\right)} \quad \ldots(2.4A)$$

where

$$\langle u \rangle = \int_0^1 u\, dz = \frac{1}{2} r \frac{M^2}{N} + \frac{R_e}{r^2}$$

is evaluated using the expression given by (2.3FF), (2.3GG) and (2.3HH).

The radial velocity distribution for $R_e^* = 1$, $R_W = 0$, 2.00 and – 2.0 and $M^2 = 0$, 16, 64, 256 are shown in fig. (2 to 4). The velocity distribution for $R_W = 0$ (fig. 4) corresponds to the source flow between non-porous disks. It is observed that the effect of injection, for small values of M, it to increase the maximum velocity at z = 0, whereas, suction has an opposite effect on the radial velocity distribution. For higher value of M(M^2 = 256). We find that for both suction and injection, the radial velocity at z = 0 has more or less the same value. We also find that the radial velocity has a uniform value in the main body of the fluid and the variation in radial velocity are confined to boundary layers near the walls. On the other hand, in the absence of magnetic field (M^2 = 0) the radial velocity distribution are parabolic for both suction and injection.

The perturbation term $f'_{0,1}$ due to interaction between the source free flow and the source flow is presented together with $f'_{1,0}$ in fig. 5. It is found that $f'_{0,1}$ is two to three orders smaller than $f'_{1,0}$ and the terms $f'_{-1,0}$ and $f'_{0,-1}$ exhibit the characteristic flattening of the distribution due to magnetic field.

The existence of an inflection point in a flow field is important for the stability consideration. Hence the conditions required for the presence of an inflexion point have been examined. The value of critical Reynolds number $R^*_{e_c}$ below which no inflexion point exists at the disks is obtained from the condition that

$$\left(\frac{\partial^2 u}{\partial z^2}\right)_{z=\pm 1} = 0$$

i.e. $$\frac{1}{2} R_W f'''_{-1}(\pm 1) + R^*_{e_c} f_0(\pm 1) + R^{*2}_{e_c} f_1(\pm 1) = 0 \qquad \ldots(2.4B)$$

Solution of (2.4B) for various values of R_w is shown in table (2.1). It is observed that for small value of M^2, $R^*_{e_c}$ is higher than the corresponding hydrodynamics values. As M^2 increases, the value of $R^*_{e_c}$ decreases. It is also observed that for injection $R^*_{e_c}$ is smaller than that of the value for flow between non-porous disks i.e. for R_w = 0. The opposite is true for suction.

Pressure Distribution :

Using equation (2.3A) and equation (2.2S) in (2.2M), the total expression for pressure distribution is

$$P(r,z) = \frac{1}{4} r^2 \frac{M^2}{N}\left(h_{-1,0} + \frac{1}{N} h_{-1,1}\right) - \frac{M^2}{N}$$
$$\left[f'_{-1,0} + \frac{1}{N} f'_{-1,1} + \frac{1}{2}\frac{M^2}{N}\left(f^2_{-1,0} + \frac{2}{N} f_{-1,0} f_{-1,1} + \frac{1}{N^2} f^2_{-1,1}\right)\right]$$
$$+ R_e\left[\left(h_{0,0} + \frac{1}{N} h_{0,1}\right)\log r + \frac{R_e}{r^2} h_{1,0}\right] + \text{Constant} \qquad \ldots(2.4C)$$

where the constant is determined from the condition that the pressure is known at a point in the flow, this can be understood by the following :

$$P(R,1) = \frac{1}{4} R^2 \frac{M^2}{N}\left(h_{-1,0} + \frac{1}{N} h_{-1,1}\right) - \frac{1}{2}\frac{M^4}{N^2}$$
$$+ R_e\left[\left(h_{0,0} + \frac{1}{N} h_{0,1}\right)\log R + \frac{R_e}{R^2} h_{1,0}\right] + \text{Constant} \qquad \ldots(2.4D)$$

since $f'_{-1,\alpha}(1) = 0 \;\; f_{-1,1}(1) = 0$ and $f_{-1,0}(1) = 1$

Hence the expression for the pressure distribution is

$$P(r,z) - P(R,1) = -\frac{1}{4}\frac{M^2}{N}\left(h_{-1,0} + \frac{1}{N} h_{-1,1}\right)$$

$$(R^2 - r^2) - \frac{M^2}{N}\Bigg[f'_{-1,0} + \frac{1}{N}f'_{-1,1}$$
$$+\frac{1}{2}\frac{M^2}{N}\left(f^2_{-1,0} + \frac{2}{N}f_{-1,0}f_{-1,1} + \frac{1}{N^2}f^2_{-1,1} - 1\right)\Bigg]$$
$$-R_e\left[\left(h_{0,0} + \frac{1}{N}h_{0,1}\right)\log\frac{R}{r} - \frac{R_e}{R^2}h_{1,0}\left(1 - \frac{R^2}{r^2}\right)\right] \quad \text{...(2.4E)}$$

The pressure drop in the radial direction at the disks is

$$P^* = P(r,1) - P(R,1) = -\frac{1}{4}\frac{M^2}{N}\left(h_{-1,0} + \frac{1}{N}h_{-1,1}\right)(R^2 - r^2)$$
$$-R_e\left[\left(h_{0,0} + \frac{1}{N}h_{0,1}\right)\log\frac{R}{r} - \frac{R_e}{R^2}h_{1,0}\left(1 - \frac{R^2}{r^2}\right)\right] \quad \text{...(2.4F)}$$

The inertia less or creeping flow pressure drop in the radial direction is obtained by neglecting the contribution form inertial term in (2.4F) and is given by

$$P^* = P(r,1), -P(R,1) = -\frac{1}{4}\frac{M^2}{N}(h_{-1,0})(R^2 - r^2) - R_e(h_{0,0})\log\frac{R}{r} \quad \text{...(2.4G)}$$

The pressure distribution is presented in fig. 6, for $R_e = 3 \times 10^4$, R = 240 and various values of R_w. Comparing our results with those of Elkouh [11], we find that even though the pressure distributions are similar there is a change in magnitude of order one due to magnetic field. The inertia terms cause a decrease in pressure in the radial direction.

Skin Friction :

The dimensionless shear stress at the upper disks is given as

$$\tau_0 = -\left(\frac{\partial u}{\partial z}\right)_{z=1} \quad \text{...(2.4H)}$$

In equation (2.4H) using the expression of u from equation (2.2N)

$$\tau_0 = -\left[\frac{1}{2}r\frac{M^2}{N}\left(f''_{-1,0} + \frac{1}{N}f''_{-1,1}\right) + \frac{R_e}{r}\left\{\left(f''_{0,0} + \frac{1}{N}f''_{0,1}\right) + \frac{R_e}{r^2}f''_{1,0}\right\}\right]_{z=1} \quad \text{...(2.4I)}$$

The inertia less shear stress at the upper disk, which is obtained by neglecting the contribution from inertia terms in (2.4I) is

$$(\tau_0)_{\text{intertialess}} = -\left[\frac{1}{2}r\frac{M^2}{N}f''_{-1,0} + \frac{R_e}{r}f''_{0,0}\right]_{z=1} \quad \text{...(2.4J)}$$

The shear stress ratio

$$\tau_0^* = \frac{\tau_0}{(\tau_0)_{\text{intertialess}}} \quad \text{...(2.4K)}$$

is evaluated for R_e^* = 0, 1, 2 and 3, R_w = 0, + 2 and – 2 and M^2 = 4, 16 and 256. The results are presented in table (2.3).

We observe that the shear ratio increases with an increase in M for a given R_e^* and R_w. It decreases with an increase in R_e^* for given M and R_w, except in the case of M^2 = 256, R_w = ± 2 the shear stress ratio first increases and then decreases. Comparing with the results of the hydrodynamic case. We find that the shear stress ratio in our case decrease very slowly with R_e^* for a given M and R_w and the incipient flow reversal (i.e., τ_0^* = 0) occurs at very large values of R_e^* for M^2 = 4, it occurs at R_e^* = 12 approximately and for M^2 = 16, it occurs at R_e^* = 25.

Table 2.1

$M^2 = 0$		$M^2 = 4$		$M^2 = 15$	
R_w	$R^*_{e_c}$	R_w	$R^*_{e_c}$	R_w	$R^*_{e_c}$
0.0	1.7219	0.0	5.5172	0.0	3.2261
0.5	1.5272	0.5	4.5993	0.5	1.2252
1.0	1.3842	1.0	3.3113		
- 0.5	1.9522	- 0.5	6.2121	- 0.5	4.6664
- 1.0	2.2077	- 1.0	6.7232	- 1.0	5.9883

Table 2.2

$M^2 = 4$			
R^*_e	$R_w = 0$	$R_w = +2$	$R_w = -2$
0	1	0.7821	1.2742
1	0.9640	0.7587	1.1831
2	0.9230	0.7230	1.0863
3	0.8428	0.6688	1.0132

$M^2 = 16$			
R^*_e	$R_w = 0$	$R_w = +2$	$R_w = -2$
0	1	0.8431	1.1541
1	0.9819	0.8219	1.1423
2	0.9511	0.7912	1.1103
3	0.9194	0.7577	1.0812

$M^2 = 256$			
R^*_e	$R_w = 0$	$R_w = +2$	$R_w = -2$
0	1	0.9352	1.0600
1	0.9984	0.9369	1.0612
2	0.9965	0.9363	1.0571
3	0.9942	0.9344	1.0543

Table 3

<u>Shear Stress Ratio</u>

$M^2 = 4$

$R_w = 0$		$R_w = +2$		$R_w = -2$	
R_e^*	τ_e^*	R_e^*	τ_e^*	R_e^*	τ_e^*
0	1	0	0.7821	0	1.2742
1	0.9640	1	0.7587	1	1.1831
2	0.9230	2	0.7230	2	1.0863
3	0.8428	3	0.6688	3	1.0132

$M^2 = 16$

$R_w = 0$		$R_w = +2$		$R_w = -2$	
R_e^*	τ_e^*	R_e^*	τ_e^*	R_e^*	τ_e^*
0	1	0	0.8431	0	1.1541
1	0.9819	1	0.8212	1	1.1423
2	0.9511	2	0.7912	2	1.1130
3	0.9194	3	0.7577	3	1.0812

$M^2 = 256$

$R_w = 0$		$R_w = +2$		$R_w = -2$	
R_e^*	τ_e^*	R_e^*	τ_e^*	R_e^*	τ_e^*
0	1	0	0.9352	0	1.0600
1	0.9984	1	0.9369	1	1.0612
2	0.9965	2	0.9363	2	1.0571
3	0.9942	3	0.9344	3	1.0543

5. CONCULSTIONS

Navier-Stokes equations have been solved for the problem of conducting laminar source flow between two infinite non-conducting disks. The solution obtained embodies the solution for the source free flow between porous disks as well as that for source flow between non-porous disks. The results of the present investigation may be summarized as follows :

(i) For small values of M fluid injection increases the maximum radial velocity while suction decreases the maximum radial velocity.

(ii) For large value of M, the magnitude of radial velocity is more or less the same in all the three cases viz., the source flow between porous disks with suction or injection and source flow between non-porous disks.

(iii) Inertia brings about a decrease in pressure drop in the radial direction, which is opposite to the effect observed in the source free flow.

(iv) The radial velocity distribution in the case of the source flow exhibit inflexion points. For small values of the imposed magnetic field the inflexion points occur at higher values of $R^*_{e_c}$ compared with hydrodynamic case. For a given R_w, as magnetic field increases, the value of $R^*_{e_c}$ decreases.

(v) For a given R_w, the value of R^*_e at which incipient flow reversal occurs increases with increase in magnetic field.

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