# Unsteady Flow with Heat and Mass Transfer of Viscous Immiscible Fluids Between Parallel Plates.

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## Abstract:

This paper deals with the unsteady flow with heat and mass transfer of two viscous immiscible fluids between two parallel plates. The partial differential equations governing the flow and heat transfer are solved analytically using two-term harmonic and non-harmonic functions in both fluid regions of the channel. Effects of physical parameters such as height ratio, viscosity ratio, conductivity ratio, Prandtl number, Eckert number, periodic frequency parameter and pressure on the velocity and temperature distributions are given and illustrated graphically.

Keywords-Unsteady flow, heat transfer, immiscible fluids, oscillatory pressure gradient

## INTRODUCTION

Magneto hydrodynamic (MHD) is the science of motion of electrically conducting fluid in presence of magnetic field. Some of these fluids include liquid metals (such as mercury, molten iron) and ionized gases known by Physicist as Plasma, an example being the solar atmosphere. The dynamo and motor is a classical example of MHD principle. The unsteady Magneto hydrodynamics (MHD) free convective flows in a horizontal channel have over the years been subjected to numerous studies. These applications include MHD power generators, MHD pumps, liquid metal cooling of reactors, Magnetic drug targeting etc. Several Scholars and Authors have contributed their quota since the study of MHD was first initiated by the Swedish electrical engineer Hannes Alfven (1942). Shercliff (1956), Sparrow and Cess (1961), Singhand Ram (1978), Abdulla (1986), Singh (1993) among others have studied several motions of these electrically conducting fluids.

Several researches have studied and have related literatures on the effect on unsteady MHD oscillatory flow of fluid in a porous channel with heat and mass transfer and chemical reaction. Das et al (2012) analysed the effect radiative heat and mass transfer on unsteady natural convection flow through a porous medium in the regime in the presence of chemical reaction. All the above studies pertain to steady flow. [8-11] have presented analytical solutions for unsteady/oscillatory two-fluid and three-fluid flow and heat transfer in a horizontal channel.

However, most problems of practical interest is unsteady. Keeping in view the wide area of practical importance of unsteady multi-fluid flows as mentioned above, it is the objective of the present study to investigate unsteady flow and heat transfer of two–fluid model in a horizontal channel.

#### **Formulation of the Problem**

The geometry Considered here is a two dimensional unsteady flow of two immiscible fluids in a horizontal parallel permeable plates,  $o \le y \le h$  extending in the Z and X direction. The region-1(Region-I) is filled with a viscous fluid having density  $\rho 1$ ,  $\mu$ -dynamic viscosity, specific heat at constant pressure P1 and the thermal conductivity K1.  $-h \le y \le 0$  and the region (Region-II) is filled with a different viscous fluid having  $\rho 2$  density  $\mu 2$ , dynamic viscosity, specific heat at constant pressure P2 and thermal conductivity K2.

#### **REGION I:**

$$\left(\rho_0 \frac{\partial U_1'}{\partial y'}\right) = \mu_1 \frac{\partial^2 U_1'}{\partial y'^2} - \frac{\partial P'}{\partial x'} \tag{1}$$

$$\rho_1 c_p \frac{\partial T_1'}{\partial t'} = k_1 \frac{\partial^2 T_1'}{\partial y'^2} + \mu_2 \left(\frac{\partial u_1}{\partial y}\right)^2 \tag{2}$$

$$\frac{\partial C_1}{\partial t} + \frac{\partial C_1}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C_1}{\partial y^2} - \frac{K_c}{Sc} C_1$$
(3)

**REGION II:** 

$$\left(\rho_0 \frac{\partial U_1'}{\partial y'}\right) = \mu_2 \frac{\partial^2 U_2'}{\partial y'^2} - \frac{\partial P'}{\partial x'} \tag{4}$$

$$\rho_1 c_p \frac{\partial T_1'}{\partial t'} = k_2 \frac{\partial^2 T_1'}{\partial y'^2} + \mu_2 \left(\frac{\partial u_1}{\partial y}\right)^2 \tag{5}$$

$$\frac{\partial C_2}{\partial t} + \frac{\partial C_2}{\partial y} = \frac{\gamma_1}{s_c} \frac{\partial^2 C_2}{\partial y^2} - \frac{K_c}{s_c} C_2$$
(6)

where u is the x-component of fluid velocity, v is the y component of fluid velocity and T is the fluid temperature. The subscripts 1 and 2 correspond to region-I and region-II, respectively. The boundary conditions on velocity are the no slip boundary conditions which required that the x-component of velocity must vanish at the wall. The boundary conditions on temperature are isothermal conditions. We also assume the continuity of velocity, shear stress, temperature and heat flux at the interface between the two fluid layers at y=0.

The hydrodynamic boundary and interface conditions for the two fluids can then be written as

$$U_{1}'(h) = 0, \ U_{2}'(-h) = 0, \ U_{1}'(h) = U_{2}'(h), \\ \mu_{1}\frac{\partial U_{1}'}{\partial y'} = \mu_{2}\frac{\partial U_{2}'}{\partial y'}aty' = 0$$
(7)

The boundary and interface conditions on the temperature for both fluids are.

$$T_1'(h) = T_{w1}', T_2'(-h) = T_{w2}', T_1'(0) = T_2'(0), k_1 \frac{\partial T_1'}{\partial y'} = k_2 \frac{\partial T_2'}{\partial y'} aty' = 0$$
(8)

The boundary and interface conditions on the concentration for both fluids are:

$$C_1'(h) = C_{w1}', C_2'(-h) = C_{w2}', C_1'(0) = C_2'(0), D_1 \frac{\partial C_1'}{\partial y'} = D_2 \frac{\partial C_2'}{\partial y'} \text{ when } y'=0$$
(9)

The equations (1) and (5) implies that  $V'_1$  and  $V'_2$  are independent of y', they are functions of time alone. Hence  $V' = V_0 (1 + \epsilon A e^{i\omega t})$  (10)

#### **REGION I:**

$$\frac{\partial U_1}{\partial t} + (1 + \epsilon e^{i\omega t}) \frac{\partial U_1}{\partial y} = \frac{\partial^2 U_1}{\partial y^2} + P - M^2 U_1 + Gr\theta_1 + GcC_1$$
(11)

$$\frac{\partial \theta_1}{\partial t} + \left(1 + \epsilon e^{i\omega t}\right) \frac{\partial \theta_1}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta_1}{\partial y^2} \tag{12}$$

$$\frac{\partial C_1}{\partial t} + \left(1 + \epsilon e^{i\omega t}\right) \frac{\partial C_1}{\partial y} = \frac{1}{s_c} \frac{\partial^2 C_1}{\partial y^2} - \frac{K_c}{s_c} C_1$$
(13)

### **REGION II:**

$$\frac{\partial U_2}{\partial t} + \left(1 + \epsilon e^{i\omega t}\right) \frac{\partial U_2}{\partial y} = \alpha_1 \xi_1 \frac{\partial^2 U_2}{\partial y^2} + \xi_1 P - \xi_1 M^2 U_2 - \alpha_1 \xi_1 K^2 U_2 + Grm_1 \theta_2 + Gc\eta_1 C_2$$
(14)

$$\frac{\partial \theta_2}{\partial t} + \left(1 + \epsilon e^{i\omega t}\right) \frac{\partial \theta_2}{\partial y} = \frac{\beta_1 \xi_1}{Pr} \frac{\partial^2 \theta_2}{\partial y^2}$$
(15)

$$\frac{\partial C_2}{\partial t} + (1 + \epsilon e^{i\omega t}) \frac{\partial C_2}{\partial y} = \frac{\gamma_1}{sc} \frac{\partial^2 C_2}{\partial y^2} - \frac{K_c}{sc} C_2$$
(16)

The boundary and interface conditions in dimensionless form are given as follows.

$$U_1(1) = 0, \ U_2(-1) = 0, \ U_1(0) = U_2(0) \quad , \frac{\partial U_1}{\partial y} = \alpha_1 \frac{\partial U_2}{\partial y} \ at \ y = 0$$
(17)

$$\theta_1(1) = 1, \ \theta_2(-1) = 0, \ \theta_1(0) = \theta_2(0), \ \frac{\partial \theta_1}{\partial y} = \beta_1 \frac{\partial \theta_2}{\partial y} \ at \ y = 0$$
(18)

$$C_1(1) = 1, C_2(-1) = 0, C_1(0) = C_2(0), \frac{\partial C_1}{\partial y} = \gamma_1 \frac{\partial C_2}{\partial y}$$
 when y=0 (19)

## **Results and Discussion**

In this section representative, flow results for oscillatory flow and heat transfer of two immiscible fluids between two parallel plates are presented and discussed for various parametric conditions. The flow governing equations cannot be solved exactly. However the closed form solutions were found considering the cosine function for frequency parameter on velocity and pressure is assumed. The solutions are depicted graphically in Figs. 1 to 6 for different values of viscosity ratio, periodic frequency parameter and pressure on the flow and thermal

conductivity ratio, Prandtl number and Eckert number on temperature field. The parameters are fixed as 1 except the varying one, Pr=0.7, Ec=0.5 and  $t_{\odot}=5$ . Figure 1 shows that velocity profiles are suppressed for large values of Schmidt number. The flow profile is large in region-II compare to region-I, and the similar effect observed for different values of Prandtl number on velocity profile as shown in Figure 2.

From Figure 3 and 4 we can observe the variation of periodic frequency parameter M and the Grashof number in the velocity profile for different values, as M increases the flow increases, the t $\omega$  increases velocity profiles is also increases in both the regions, since the solutions are approximated by function of since the solutions are approximated by exponential function of t . $\omega$  Keeping in view the physical model of the flow of two immiscible fluids such as water and oil in petroleum industries, a study is made to know the effect of pressure on the flow as shown Fig.5.

We have considered different values of pressure for two fluids separately. For positive values of pressure on upper and lower fluids, the flow is promoted. For positive values of pressure in the lower region and negative values of pressure in the upper region display the effect of maximum velocity in region-I. On the other hand, if we take negative values of pressure in lower region and positive values of pressure in the upper region also show the maximum velocity in region-I itself but the flow direction is opposite. Assigning negative values of pressure also shows the similar effect to that for positive values of pressure except in opposite direction. It is observed that controlling the pressure parameter one can also control the direction of flow, which has immense applications in flow reversal problems. The effect of Gc is depicted in Fig.5 As the ratio increases the magnitude of suppression is large in region-I compared to region-II. This is obvious because the upper plate is maintained at a low temperature compared to region-II. Figures 6 display the effect of Pressure gradient on velocity the field. It is seen that velocity increases with increase in Pressure for different values. Thus, one can conclude that the flow can be controlled by considering different fluids having different viscosities, periodic frequency and applying different pressures.



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